Winning a game by halftime

Topics covered: Probability.

Key words: Addition rule of probability. Independent events.

Consider a sporting event that is divided into two halves, with the final score for a team being the total of the scores for each half. Examples of such events include soccer, football and basketball. Assume that the two teams are precisely evenly matched. Also, assume that the result of play in the first half does not influence play in the second half; that is, teams don't play harder because they're behind, or give up because they're behind. What is the probability that the team that is ahead at halftime will win the game?

Let's call the two teams "A" and "B." We also will ignore here the possibility that A and B have equal scores at the end of each half (i.e., a "tie"). This simplified argument is least applicable to soccer (where the total number of goals scored is typically below five), and most applicable to basketball (where the total number of points scored is typically between 50 and 120). One intuitive answer to our question would seem to be .5, with the argument being that if team A, say, happens to be ahead by 5 points at the end of the first half (i.e., they "win" the first half), since the two teams are evenly matched, it's just as likely that team B will be ahead by more than 5 points for the second half (i.e., they "win" the second half), and thus wins the game. Is this the correct way to look at the problem?

Let's write out the possible outcomes of a game. In the table below, "A" means that team A won the half, while "B" means that team B won the half. The four possible outcomes of the game can then be summarized as follows:

Who won the half? First half	Second half
A	A
A	B
B	A
B	B

Since the teams are evenly matched, the probability of A or B winning either half is .5 (in fact, this could be taken as the definition of teams being evenly matched). Since the results of the first and second halves are independent, the probability of each of the four possibilities above is .25.

(1) When either team A wins both halves, or team B wins both halves, clearly they have won the game, so the results [A, A] and [B, B] (i.e., the first and last rows of the table) are consistent with the team that is ahead at halftime winning the game.

(2) Suppose team A wins the first half and team B wins the second half (i.e., situation [A, B] occurs). Since teams A and B are evenly matched, it is equally likely that the difference favoring A in the first half will be larger than the difference favoring B in the second half, as that it will be smaller than the difference favoring

B in the second half. That is, given that [A, B] occurs, half of the time A will win the game, and half of the time B will win the game.

(3) The same argument as in (2) applies if situation [B, A] occurs. That is, given that [B, A] occurs, half of the time A will win the game, and half of the time B will win the game.

By applying the rules for calculating probabilities, we have that

$$
\begin{aligned}
&P(\text{Team ahead at halftime wins the game}) \\
&\quad = P([\mathrm{A},\mathrm{A}]) + P([\mathrm{B},\mathrm{B}]) + \left(\frac{1}{2}\right) P([\mathrm{A},\mathrm{B}]) + \left(\frac{1}{2}\right) P([\mathrm{B},\mathrm{A}]) \\
&\quad = .25 + .25 + \left(\frac{1}{2}\right)(.25) + \left(\frac{1}{2}\right)(.25) \\
&\quad = .75
\end{aligned}
$$

That is, when two evenly matched teams play, the team that is ahead at halftime can be expected to ultimately win the game 75% of the time. Note that this result has nothing to do with teams giving up at halftime, or becoming overconfident, or any other possibility of this type, since we have assumed that the first and second half results are independent. If such factors were important, we might expect the actual proportion of teams that win when ahead at halftime to be different from .75 (for example, perhaps it would be larger, if you think being ahead gives a psychological advantage to a team). Another reason that the actual proportion might be larger than .75 is if there was great disparity in quality of teams, since then the probability of one team winning both halves is greater (can you prove that the probability that one team wins both halves is minimized when the two teams are evenly matched?). Professor Hal Stern of Harvard University, in a 1994 article in *Journal of the American Statistical Association*, examined 493 National Basketball Association games from January to April 1992, and found that in 74.8% of the games, the team that was ahead at halftime ultimately won the game.

Professor Stern extended this model to allow investigation of this effect at points other than halftime. His model implies that the probability that the team that is ahead after one quarter wins the game is $\frac{2}{3}$, while the probability that the team that is ahead after three quarters wins the game is $\frac{5}{6}$. These can be compared to the observed proportions in his data of .667 and .821, respectively.

Summary

In a sporting event between two evenly matched teams the team that is ahead at halftime can be expected to ultimately win the game 75% of the time. This calculation assumes that ties in score do not occur and that the result of play in the first half does not influence play in the second half.