

Amplitude Scaling of Asymmetry-Induced Transport

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Abstract. Our initial experiments on asymmetry-induced transport in non-neutral plasmas found the radial particle flux at small radii to be proportional to ϕ_a^2 , where ϕ_a is the applied asymmetry amplitude. Other researchers, however, using the global expansion rate as a measure of the transport, have observed a ϕ_a^1 scaling when the rigidity (the ratio of the axial bounce frequency to the azimuthal rotation frequency) is in the range one to ten. In an effort to resolve this discrepancy, we have extended our measurements to different radii and asymmetry frequencies. Although the results to date are generally in agreement with those previously reported (ϕ_a^2 scaling at low asymmetry amplitudes falling off to a weaker scaling at higher amplitudes), we have observed some cases where the low amplitude scaling is closer to ϕ_a^1 . Both the ϕ_a^2 and ϕ_a^1 cases, however, have rigidities less than ten. Instead, we find that the ϕ_a^1 cases are characterized by an induced flux that is comparable in magnitude but opposite in sign to the background flux. This suggests that the mixing of applied and background asymmetries plays an important role in determining the amplitude scaling of this transport.

INTRODUCTION

Malmberg-Penning traps are now being used in a variety of experiments. Understanding the properties of these traps is thus of considerable practical as well as fundamental interest. It has long been known that confinement in Malmberg-Penning traps is limited by the presence of asymmetric electric and magnetic fields. Early confinement studies found that at low neutral pressures the confinement time was much less than expected from transport due to electron-neutral collisions [1]. It was suggested that this anomalous transport was due to the presence of electric or magnetic fields that break the cylindrical symmetry of the trap. The presence of such asymmetries would produce a radial component to the $E \times B$ drift that would lead to particle loss to the walls of the trap. This suggestion led to a number of experiments [2-8] employing *applied* asymmetries in order to study the transport in a controlled manner. Most of the experiments have used electric asymmetries since these are easily applied and manipulated using the sector wall portions of the confinement region of the trap.

A basic issue of asymmetry-induced transport is the scaling of the transport with asymmetry amplitude. Current theory [9] predicts two transport regimes depending on the amplitude of the asymmetry in the plasma ϕ . For smaller amplitudes (the plateau regime), the radial flux is proportional to the square of the asymmetry potential ϕ^2 . For larger amplitudes (the banana regime), the flux scales like the square root of the asymmetry potential $\phi^{1/2}$.

Experiments on amplitude scaling have used a variety of measures to characterize the transport and have applied the asymmetries in different ways. Notte and Fajans [3] applied a switched DC voltage to a 50 degree wall patch in the central part of the confinement region. Following earlier work, they measured the time τ_m for the central density to decrease by one-half and found that this confinement time scaled like ϕ_a^{-m} with $1.7 < m < 2.1$. This scaling roughly agrees with plateau-regime scaling, although the voltages used in this experiment (up to 40 volts) would seem to be much too high to satisfy the plateau regime requirement.

More recently, Kriesel and Driscoll [6-8] have employed switched DC voltages applied at the end of the plasma column to study this transport over a wide range of parameters. For most of these experiments they used the rate of change of the mean-square-radius $\Delta\nu$ (with the background rate subtracted off) to characterize the transport. They found two transport regimes determined by the experimental value of the rigidity, which is the ratio of the axial bounce frequency to the azimuthal rotation frequency. When the rigidity was between one and ten, the expansion rate $\Delta\nu$ scaled like ϕ_a^1 , whereas higher values of rigidity gave a ϕ_a^2 scaling.

Our earlier studies [5] in a modified trap (described below) found that the time rate of change of the plasma density dn/dt (measured at one radius) scaled like $\phi_a^{2.1}$ for small asymmetry amplitude and like $\phi_a^{1.3}$ for larger amplitudes. The radial flux Γ was found to have a similar scaling. The rigidity for these measurements was small (approximately 0.2), and our ϕ_a^2 scaling thus seemed to contradict the result of Kriesel and Driscoll. The expansion rate $\Delta\nu$, however, is related to a radial integral [7] of the particle flux Γ , so the contradiction could not be established by measurements of Γ at a single radius. We have thus expanded our range of measurements to resolve this issue. Although the results are generally in agreement with those previously reported, we have observed some cases where the low amplitude scaling is closer to ϕ_a^1 . Both the ϕ_a^2 and ϕ_a^1 cases, however, have rigidities less than ten. Instead, we find that the ϕ_a^1 cases are characterized by an induced flux that is comparable in magnitude but opposite in sign to the background flux.

EXPERIMENTAL RESULTS

Our experiments are performed in the modified Malmberg-Penning trap shown in Figure 1. The usual plasma column is replaced by a biased wire running along the axis of the trap. Electrons injected into this device have the same dynamical motions as those in a normal non-neutral plasma (i.e. axial bounce and azimuthal drift motions), but collective contributions to the asymmetry potential are largely

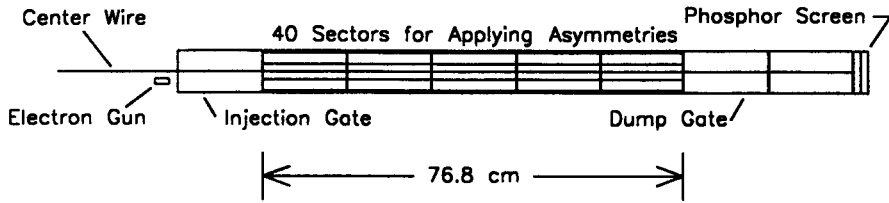


FIGURE 1. Schematic of the Occidental Trap. The plasma is replaced by a biased wire that maintains the basic dynamical motions of the injected electrons. Forty wall sectors allow for the application of asymmetries consisting of essentially one Fourier mode.

eliminated since the lower density (10^5 cm^{-3}) and higher temperature (4 eV) of the electrons give a Debye length larger than the trap radius. The asymmetry potential within the confinement region is thus essentially the vacuum potential and we need not be concerned with plasma modifications of the asymmetry potential. Despite these changes in the plasma parameters, the confinement time scaling with no applied asymmetries [10] shows the same $(L/B)^2$ dependence found in higher density experiments [11,12], thus supporting the notion that the asymmetry-induced transport is a single particle effect.

For the current experiments, forty wall sectors are employed to produce the applied asymmetry. By judiciously selecting the voltage applied to each sector we can produce an asymmetry consisting of essentially a single Fourier mode, thus eliminating the sum over modes in the transport theory [9] and making for a simpler comparison between theory and experiment. We use AC voltages at a variable frequency f since this gives us an additional experimental parameter that can be varied independently of other quantities. Electrons injected into the trap are quickly dispersed into an annular distribution [13]. At the end of an experimental cycle the electrons are dumped onto a phosphor screen and the resulting image is digitized with a cooled CCD camera. A radial cut through this image gives the axially-integrated density profile $n(r, t)$ of the electrons. For this study it is then useful to numerically integrate the density profile to obtain the quantity $g(r, t)$:

$$g(r, t) = -\frac{1}{r} \int_0^r r dr \cdot n(r, t)$$

The time derivative of $g(r, t)$ then gives the radial particle flux $\Gamma(r, t) = dg/dt$.

Figure 2 shows a typical set of data. The quantity $g(r, t)$ is plotted vs time for various asymmetry amplitudes. A line is then fit to the initial slope of these curves to obtain the radial flux. Note that, at the higher amplitudes and later times, the slope changes as the initial plasma is modified by the transport. Plotting the data in this way allows us to note and avoid these saturated cases. Also note that the flux is not zero for zero applied asymmetry, i.e. there is a nonzero background flux Γ_0 . Finally, note that because we have an annular plasma both positive and negative fluxes are observed.

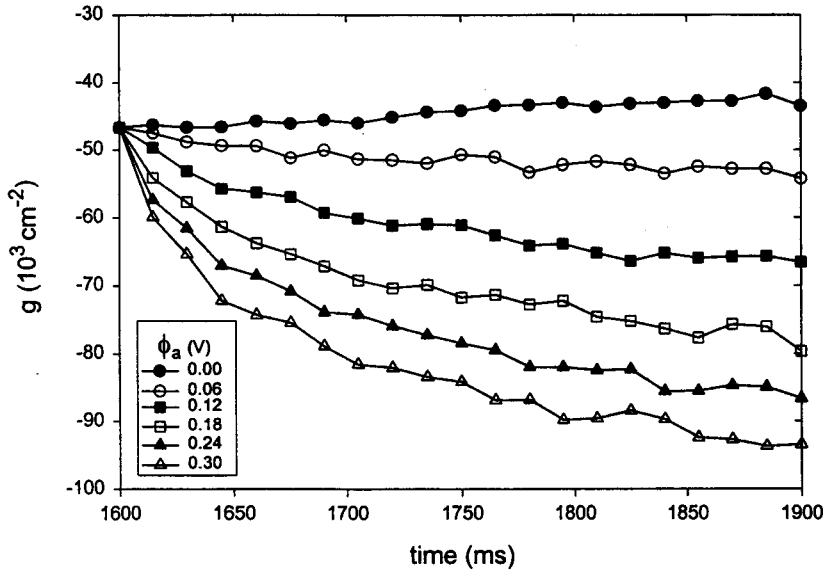


FIGURE 2. A typical data set showing the computed quantity $g(r, t)$ versus time with asymmetry amplitude as a parameter. The slope of these curves gives the radial flux $\Gamma = dg/dt$.

Figure 3 shows the magnitude of the net flux $\Delta\Gamma = |\Gamma_{\text{induced}} - \Gamma_0|$ vs asymmetry amplitude with radial position as a parameter. The experimental conditions are similar to those of reference [5], figure 3 (center wire bias = -80V, $B = 365\text{G}$, $f = 1\text{MHz}$). The log-log plot allows us to determine the scaling exponent m , where $\Delta\Gamma \propto \phi_a^m$. Typical error bars are shown, and lines of slope one and two are drawn for comparison to the data. The results are similar to those reported in reference [5]: the net flux scales like ϕ_a^2 for low amplitude asymmetries, in agreement with plateau-regime theory. At higher amplitudes, the scaling exponent falls off to a smaller value in the range 0.5 to 1.4. This may indicate a transition to banana-regime scaling, but the data is not sufficient to support this conclusion. Note that there is no dramatic dependence of m on radius, so if we were to calculate the expansion rate $\Delta\nu$ as in reference [8] the low amplitude scaling would still be ϕ_a^2 .

Further exploration of our parameter space has found some cases showing scaling closer to ϕ_a^1 and some of these cases are shown in Figure 4. For these parameters (as above except $f = 0.35\text{MHz}$), the ϕ_a^1 cases occur at smaller radii while ϕ_a^2 cases occur at larger radii, so, again, a calculation of expansion rate would not yield a ϕ_a^1 scaling.

Under what conditions do we obtain fluxes with a ϕ_a^1 scaling? In Figure 5 we plot the scaling exponent m versus the rigidity for the cases we have considered. Note that all these cases have rigidity less than ten (the borderline value for the Kriesel/Driscoll regimes), and, more importantly, that the same value of rigidity

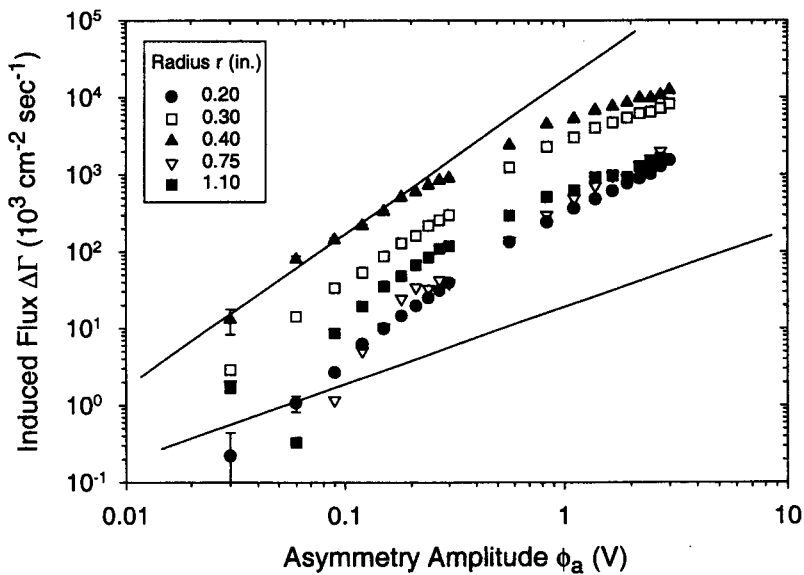


FIGURE 3. Log-log plot of the net asymmetry-induced flux $\Delta\Gamma$ vs. the applied asymmetry amplitude at the wall ϕ_a . The asymmetry frequency f is 1MHz. Lines of slope one and two are shown for comparison. For the cases shown the low amplitude scaling is close to ϕ_a^2 .

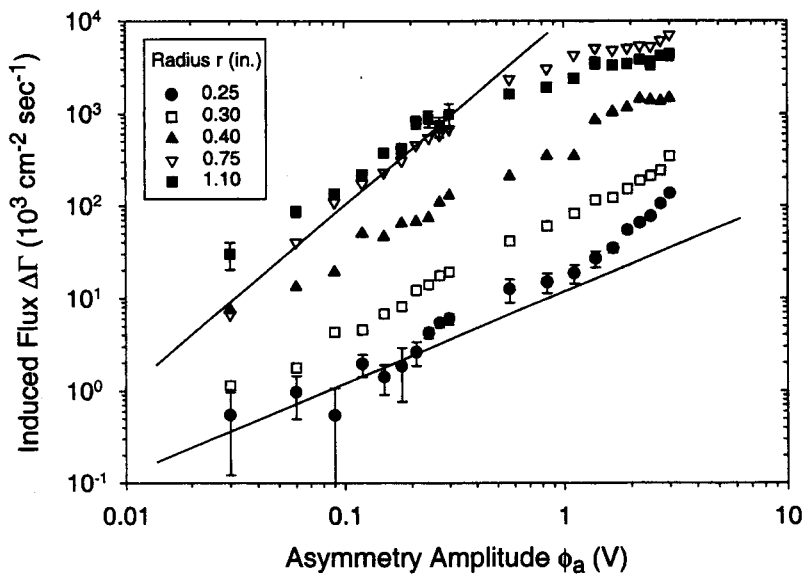


FIGURE 4. Log-log plot of $\Delta\Gamma$ vs. ϕ_a for $f = 0.35\text{MHz}$. For the lower radii the scaling exponent is close to one.

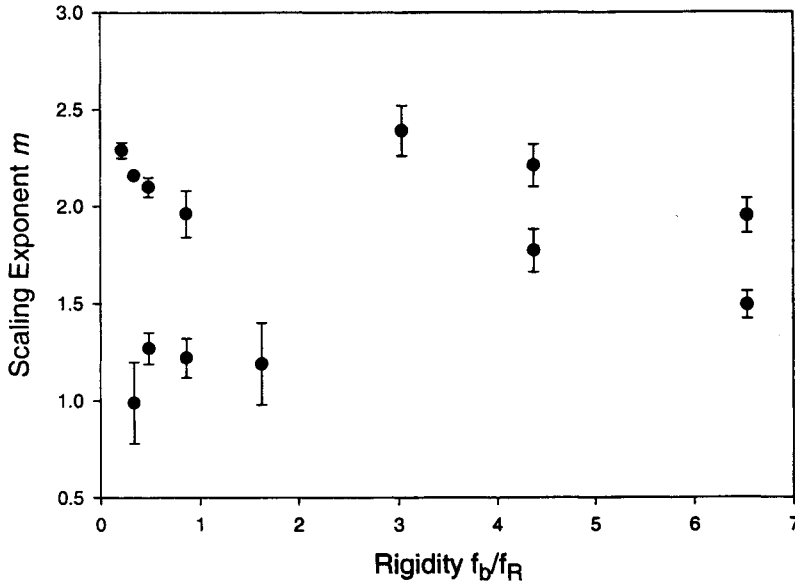


FIGURE 5. Scaling exponent m vs. rigidity. No correlation is observed with this parameter.

gives both $m \approx 1$ and $m \approx 2$ cases. Thus, in our experiment the scaling exponent is not correlated with rigidity.

We have tried to find correlation with other parameters as well. Since our experiments use a nonzero asymmetry frequency f , it might make sense that the rigidity should be modified from f_b/f_R to $f_b/(f - f_R)$ since presumably this ratio measures rigidity relative to the asymmetry. A plot of the data, however, shows no correlation of the scaling exponent with this modified rigidity either. Other quantities that fail to correlate with m include $f - f_R$, $(f - f_R)/f_R$, Γ_0 , and Γ_{induced} .

A re-examination of the original data for the $m = 1$ cases shows typical g vs t plots like the one shown in Figure 6. For these cases, the background flux is comparable in magnitude to the induced flux. Following this clue, we have plotted in Figure 7 the scaling exponent vs the ratio of the background flux to the induced flux at a typical asymmetry value of 0.3V. The correlation here is good, with $m = 2$ cases corresponding to smaller values of $\Gamma_0/\Gamma_{0.3V}$ and the $m = 1$ cases corresponding to larger negative values.

DISCUSSION

The fact that our $m = 1$ cases occur when the background flux is comparable in magnitude to the induced flux suggests a simple mixing of applied and background asymmetries. Suppose that the true flux scaling is $\Gamma \propto \phi_a^2$ and that when both

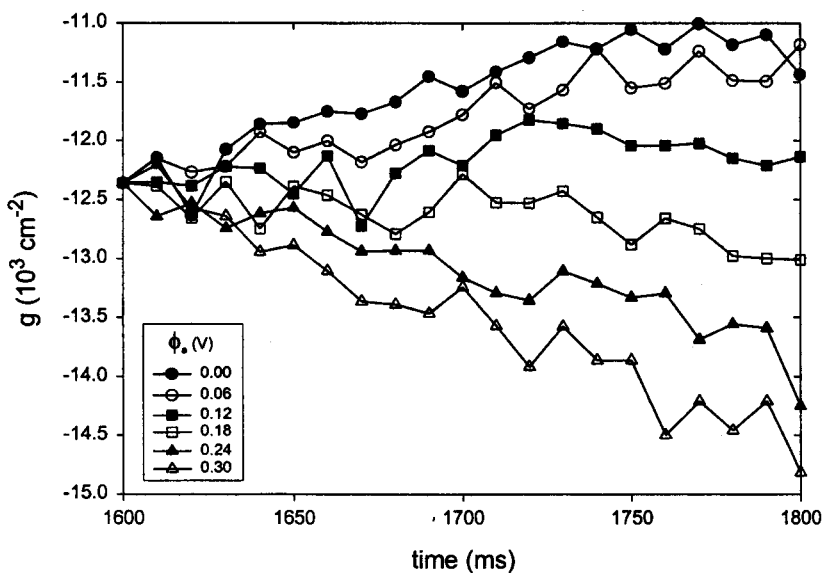


FIGURE 6. Typical g vs. t data for cases yielding a scaling exponent near one.

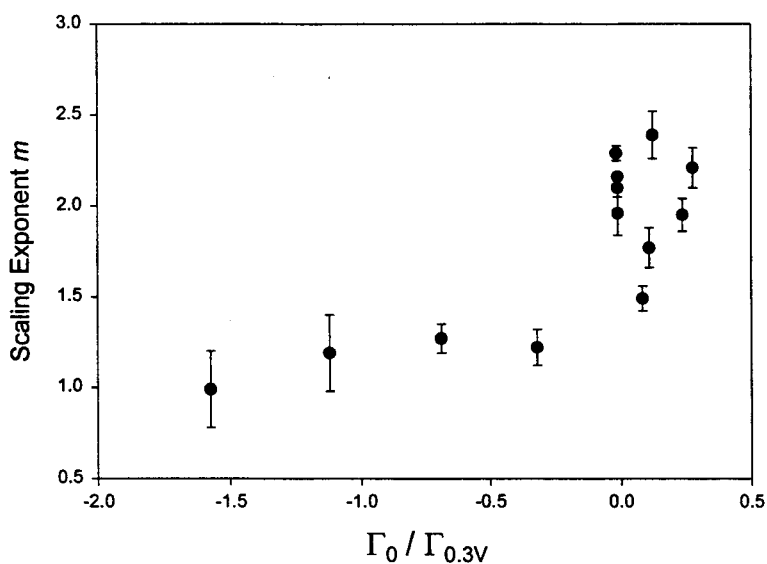


FIGURE 7. The scaling exponent shows a correlation with the ratio of the background flux Γ_0 to a typical induced flux $\Gamma_{0.3V}$

a background asymmetry ϕ_0 and an applied asymmetry ϕ_a are present they add directly. The total asymmetry squared is then $\phi_a^2 + 2\phi_a\phi_0 + \phi_0^2$ and if $\phi_a \ll \phi_0$, we obtain $\Gamma - \Gamma_0 \propto \phi_a$. Such a model, however, is not consistent with the details of the experimental data or with the theory. At least one experimental case where the flux scales like ϕ_a^1 occurs in the middle of our plasma with ϕ_a^2 cases at smaller and larger radii. The above model would then require a background asymmetry that is peaked in the middle of the plasma and it is hard to imagine how such an asymmetry could be produced. This direct addition model is also inconsistent with the theory, which says the flux is given by a sum of terms, one for each Fourier mode produced by the asymmetries. In our case, the background asymmetry is static (i.e. zero frequency) while the applied asymmetry has a nonzero frequency. These will necessarily give different Fourier modes that should not add directly. It would seem that the explanation for our observed correlation must be at a deeper level than simple addition of asymmetry potentials.

It is interesting to ask whether the Kriesel/Driscoll results can be explained in terms of our findings. From the data in reference [7], it appears that the ϕ_a^1 scaling does occur for cases where the induced transport is comparable to the background transport. The ϕ_a^2 cases are harder to judge because the background was so small it was declared to be zero. Although this would fit into our scheme (i.e. $\Gamma_0/\Gamma_{\text{induced}}$ would be zero), a more careful measurement would be required to settle this question.

It has been suggested [14] that the ϕ_a^1 scaling is due to the large size of the asymmetries employed in some experiments. In these cases the small-perturbation assumption of the quasi-linear theory is violated. If the applied asymmetry is large enough ($e\phi \gg kT$), particles will be excluded from (or pulled into) the vicinity of the biased wall sector. The perturbation of the distribution function δf would thus be large and saturated (i.e. no longer dependent on the perturbed potential in the plasma $\delta\phi$). Since the transport goes like the product $\delta f \delta\phi$ the dependence would be linear in the asymmetry amplitude. While this model might explain some experimental results (for example, the fall-off of our scaling exponent with asymmetry amplitude), it does not seem consistent with the bulk of the data, either in our experiments or those of others. For example, when we observe ϕ_a^1 scaling, it is observed even at very low asymmetry amplitudes. In contrast, Notte and Fajans observed ϕ_a^2 scaling even at extremely high asymmetry potentials. It is also not understood why this model should depend on rigidity, so it cannot explain the two scaling regimes observed by Kriesel and Driscoll.

CONCLUSION

We find that in most cases the asymmetry-induced transport in our experiment produces a net radial flux that scales like the square of the applied asymmetry amplitude for small asymmetries and falls off to a weaker dependence ($m = 0.5 - 1.4$) for larger amplitudes. We have found some cases that give a low-amplitude

scaling exponent closer to one, and these cases occur when the induced flux is comparable in magnitude but opposite in sign to the background flux. We do not yet understand the reason for this correlation.

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