Experimental Test of the Resonant Particle Theory of Asymmetry-Induced Transport

D.L. Eggleston

Occidental College, Physics Department, Los Angeles, CA 90041

Abstract. While it is easy to experimentally demonstrate that applied field asymmetries produce radial transport, convincing comparisons of experiment and theory have yet to be made. A key prediction of the theory is that the transport will be dominated by particles that move in resonance with the asymmetry. For the general case of a time-varying asymmetry, the resonance condition is $\omega - l\omega_R - kv = 0$, where v is the axial velocity, ω_R is the $E \times B$ rotation frequency, and ω, l and k are the asymmetry frequency, azimuthal and axial wavenumbers, respectively. We present experiments on our low density trap in which ω, ω_R, and k are varied and the resulting radial particle flux is measured. The experiments show a resonance in the flux similar to that predicted by theory. The peak frequency of this resonance increases with ω_R and k, but not in the way theory predicts. The peak magnitude of the measured transport is roughly forty times smaller than the theoretical prediction, and low-frequency asymmetries are especially ineffective at producing transport.

INTRODUCTION

Plasma traps of the Malmberg-Penning type have been found to be useful in a variety of fields including basic plasma physics, atomic spectroscopy, anti-matter physics, and mass spectroscopy. Early studies of the confinement time of such traps found good agreement between experiments [1] and a transport theory [2] based on collisions with neutrals. However, at the lowest neutral pressures the confinement time was much lower than expected [3] and decreased with machine length [4]. It was suggested that this anomalous transport was due to the presence of electric or magnetic fields that break the cylindrical symmetry of the trap. The presence of such asymmetries would produce a radial component to the $E \times B$ drift that would lead to particle loss. This notion was later supported by further confinement studies [5] as well as experiments with applied asymmetries [6–8].

These early papers also suggested that the asymmetry-induced transport might be described by a theoretical model developed in early studies of radial transport in tandem mirrors [9–13] where static asymmetric end cells produced radial grad-

CP498, *Non-Neutral Plasma Physics III*, edited by John J. Bollinger, et al.

B drifts that largely determined the radial particle flux. A key prediction of the theory is that the resulting transport will be dominated by particles whose axial bounce motion and azimuthal drift motion causes them to move in resonance with the asymmetry. As these resonant particles repeatedly encounter the asymmetry they take radial steps in the same direction, thus allowing them to diffuse more quickly than non-resonant particles.

We have recently adapted this theory to Malmberg-Penning traps [14] and in this paper present our first attempts to test the theory using an experimental device specifically designed for the task. While the experiments provide evidence for the dominance of resonant particles they also contradict other predictions of the theory.

ASYMMETRY-INDUCED TRANSPORT THEORY

The geometry of the non-neutral experiments is cylindrical with an axial magnetic field B. The magnetic field is typically strong enough that the Larmor radius is much smaller than any other scale length in the plasma and all relevant frequencies are small compared to the cyclotron frequency. Asymmetric electric fields are applied by placing voltages on wall sectors. Under these conditions the basic equations for a non-neutral plasma are Poisson's equation, the drift kinetic equation with a collision operator, and the boundary conditions on the conducting walls. For simplicity we take as our model a plasma of length L with flat ends, thus ignoring end effects. This allows us to linearize the potential as $\phi(r,\theta,z,t) = \phi_0(r) + \phi_1(r,\theta,z,t)$ where

$$\phi_1(r,\theta,z,t) = \sum_{n,l,\omega} \phi_{nl\omega}(r) \cdot \exp\left\{ i\left(\frac{n\pi}{L}z + l\theta - \omega t\right)\right\} \tag{1}$$

and similarly for the distribution function f. For an electron plasma ($q = -e$) Poisson's equation then becomes

$$\left[\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - \frac{l^2}{r^2} - \left(\frac{n\pi}{L}\right)^2\right]\phi_{nl\omega}(r) = 4\pi e \int dv \frac{\frac{cl}{rB}\frac{\partial f_0}{\partial r} - \frac{n\pi}{L}\frac{e}{m}\frac{\partial f_0}{\partial v}}{\frac{n\pi}{L}v + l\omega_R - \omega}\phi_{nl\omega}(r) \tag{2}$$

where ω_R is the azimuthal $E \times B$ rotation frequency of the plasma column, $\phi_{nl\omega}(r)$ is the Fourier amplitude of the asymmetry mode characterized by axial mode n, azimuthal mode l, and frequency ω, and the integral is over the axial velocity v.

The form of the resulting radial particle flux depends on the relative size of an effective collision frequency ν_{eff} and the oscillation frequency ω_T of particles trapped in the asymmetry potential, where $\nu_{eff}^3 \approx \nu_{ee}\left(\frac{n\pi\bar{v}}{L}\right)^2$ and

$$\omega_T^2 = \left\{\frac{e}{m}\left(\frac{n\pi}{L}\right)^2 - \frac{cl^2}{rB}\frac{d\omega_R}{dr}\right\}\phi_{nlw}. \tag{3}$$

When $\nu_{eff} >> \omega_T$, frequent collisions interrupt the trapped particle orbits and the basic radial step is the radial drift velocity times the time between collisions. Deviations from unperturbed orbits are small and a perturbation approach is appropriate. This is called the resonant plateau regime. When $\nu_{eff} < \omega_T$, a trapped particle can complete at least one oscillation before a collision knocks it out of resonance. Now the basic radial step is the radial extent of the drift during a trapping oscillation and the orbits are fully nonlinear. A heuristic derivation of the resulting radial flux is often employed for this so-called banana regime. The resulting radial particle flux for the plateau regime is given by (See reference [14] for details)

$$\Gamma_{plateau} = -\sum_{n,l,\omega} \frac{n_0}{\sqrt{2\pi\overline{v}^2}} \frac{L}{|n|} \left|\frac{cl\phi_{nl\omega}}{rB}\right|^2 \left[\frac{1}{n_0}\frac{dn_0}{dr} + \sqrt{2}\frac{n\pi}{L}\frac{r\omega_c}{l\overline{v}}x\right] e^{-x^2} \tag{4}$$

and for the banana regime by

$$\Gamma_{banana} = -\sum_{n,l,\omega} \frac{n_0}{\sqrt{2\pi}} \frac{\nu_{ee}\left(\frac{L}{n\pi}\right)^2\left(\frac{l\overline{v}}{r\omega_c}\right)^2\left(\frac{e\phi_{nl\omega}}{T}\right)^{1/2}}{\left\{1-\left(\frac{lL}{n\pi}\right)^2\frac{1}{r\omega_c}\frac{d\omega_R}{dr}\right\}^{3/2}} \left[\frac{1}{n_0}\frac{dn_0}{dr} + \sqrt{2}\frac{n\pi}{L}\frac{r\omega_c}{l\overline{v}}x\right] e^{-x^2}. \tag{5}$$

For simplicity we have assumed here that the temperature T is constant with radius. The variable x is equal to $v_{res}/\sqrt{2}\overline{v}$, where $v_{res} = \frac{L}{n\pi}(\omega - l\omega_R)$ is the resonant velocity for the asymmetry mode n, l, ω. The symbols $\overline{v}$, ω_c, and ν_{ee} are the thermal velocity, the cyclotron frequency, and the electron-electron collision frequency, repectively.

It is worth noting several features of these solutions. Both plateau and banana regime fluxes involve a sum over all the asymmetry modes produced by the wall voltages. The square brackets contain a diffusive term $\frac{1}{n_0}\frac{dn_0}{dr}$ and a generalized mobility $\sqrt{2}\frac{n\pi}{L}\frac{r\omega_c}{l\overline{v}}x$ (note that this latter term reduces to eE/kT for $\omega = 0$). The plateau regime flux is independent of the collision frequency and is proportional to the square of the asymmetry amplitude, whereas the banana regime flux depends linearly on ν_{ee} and scales like $\phi_{nl\omega}^{1/2}$. The dominance of the flux by resonant particles is reflected in the e^{-x^2} factor which stems from evaluating the Maxwellian distribution function at the resonant velocity. Note that x can be positive or negative as ω is greater than or less than ω_R. Thus, while static field asymmetries $(\omega = 0, x < 0)$ move electrons radially outward $(\Gamma > 0)$, an appropriately chosen asymmetry $(\omega > \omega_R, x > 0)$ can move particles radially inward as is observed in "rotating wall" experiments [6,8]. Here we use the convention that $\omega > 0$ corresponds to an asymmetry that rotates with the plasma column and $\omega < 0$ to one that rotates against the column.

The presence of ω in the variable x provides the experimentalist with an ideal way of testing the notion that resonant particles dominate the transport. By varying ω one can obtain any value of the resonant velocity v_{res} and the resulting flux should exhibit a resonance as v_{res} sweeps through the distribution function. However this

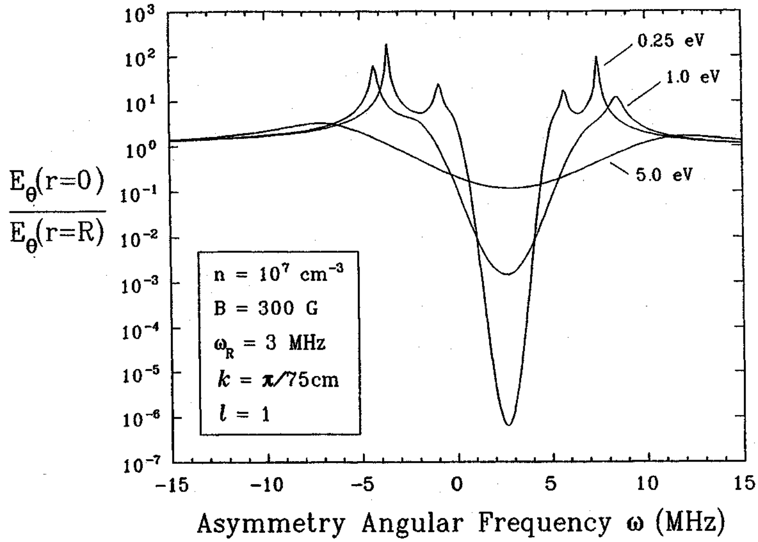

FIGURE 1. Computed variations of the normalized E_θ at the plama center versus asymmetry frequency ω. The three curves correspond to three plasma temperatures. The strong variations in E_θ are produced by plasma collective effects and make it difficult to observe the resonant particle dominance of the radial transport.

approach is complicated by the strong ω-dependence of the asymmetry potential $\phi_{nl\omega}$. Figure 1 shows numerical solutions of Equation (2) for typical plasma parameters [14]. We plot $E_\theta = l\phi_{nl\omega}/r$ at the center of the plasma (normalized to its value at the wall) as a function of asymmetry frequency ω. Note that E_θ varies by many orders of magnitude as adjustments of ω produce plasma phenomena ranging from standing waves (the peaks of the curves) to Debye shielding (the strong dip around $\omega = \omega_R$). These variations in E_θ (and thus in the flux Γ) tend to dominate or mask those produced by resonant particle effects. This produces, for example, enhanced transport when the asymmetry is at a standing wave frequency of the plasma column [6]. Nonlinear collective processes are also possible [15]. These collective effects, although interesting, are not, in our view, essential to the transport physics. We note, then, that the variations in E_θ are reduced as the temperature is increased and/or the density is reduced (see reference [14]).

These considerations led us to the modified trap design shown in Figure 2. The plasma is replaced by a biased wire running along the axis of the trap. Electrons injected into this device have the same dynamical motions as those in a normal non-neutral plasma (i.e. axial bounce and azimuthal drift motions), but the collective variations of $\phi_{nl\omega}$ are eliminated since the lower density ($10^5\ cm^{-3}$) and higher temperature (4 eV) of the electrons give a Debye length larger than the trap radius. Despite these changes, the confinement time scaling with no applied asymmetries

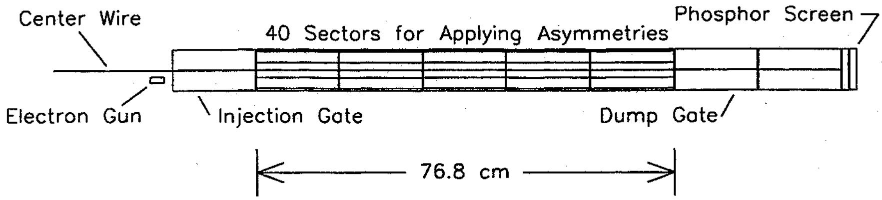

FIGURE 2. Schematic of the Occidental Trap. The plasma is replaced by a biased wire that maintains the basic dynamical motions of the injected electrons. Forty wall sectors allow for the application of asymmetries consisting of essentially one Fourier mode.

[16] shows the same $(L/B)^2$ dependence found in higher density experiments [4], thus supporting the notion that the transport is a single particle effect.

For the current experiments, up to forty wall sectors are employed to produce an asymmetry consisting of a single Fourier mode, thus eliminating the sum over n, l, and ω in the flux and making for a simpler comparison between theory and experiment. Electrons injected into the trap are quickly dispersed into an annular distribution [17]. At the end of an experimental cycle the electrons are dumped onto a phosphor screen and the resulting image is digitized. A radial cut through this image gives the density profile of the electrons. Profiles are taken both with the asymmetry on and off, and the change in density $\delta n(r)$ is either used directly to approximate dn/dt or integrated to give the radial particle flux $\Gamma(r)$.

EXPERIMENTAL RESULTS

Our initial data addresses three aspects of the theory: 1) the scaling of transport with asymmetry amplitude, 2) the dominance of the transport by resonant particles and 3) the absolute magnitude of the transport flux. Figure 3 shows the scaling of dn/dt with the amplitude of the asymmetric potential applied to the wall. The scaling is consistent with plateau regime theory (i.e. ϕ^2) when the amplitude is small and falls off to roughly $\phi^{4/3}$ at higher amplitudes. The banana regime scaling of $\phi^{1/2}$ is not observed.

Figure 4 shows the radial flux vs. asymmetry frequency at three radial positions. The radial density profile is shown in the inset. The data is qualitatively consistent with resonant particle theory. When the density gradient is large, the flux should go like e^{-x^2}, a Gaussian curve centered where $\omega = \omega_R$. This behavior is shown by the curves for r/R equal to 0.28 and 0.56 (note that ω_R is set by the center wire bias and decreases with radius). At the top of the density profile the gradient is zero, so we expect an xe^{-x^2} behavior, and this seems to match the $r/R = 0.39$ curve. Although not shown, we have verified that the curves shift horizontally in an appropriate way as the center wire bias (and thus ω_R) is varied. Also, if the asymmetry is made to spin opposite the direction of ω_R (corresponding to negative

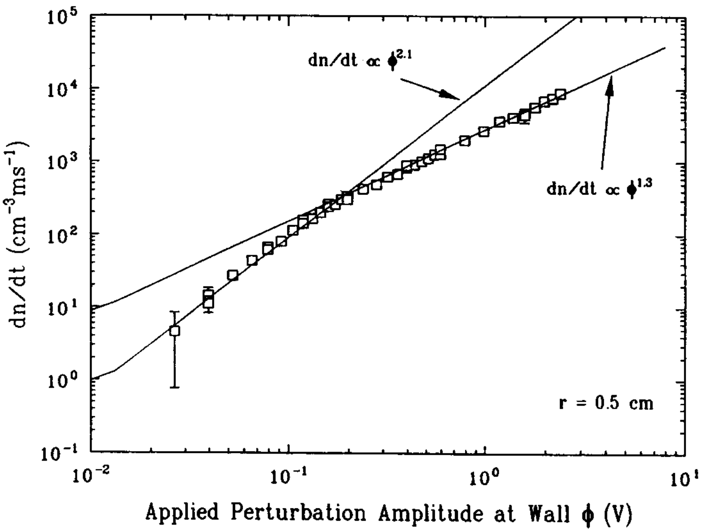

FIGURE 3. Log-log plot of the asymmetry-induced rate of density change dn/dt versus the asymmetry amplitude at the wall. The scaling is consistent with plateau-regime theory only for low amplitudes.

values of ω), no resonances are observed in the flux.

Figure 5 shows how the peak frequency of these flux resonances varies with radius and axial mode number n, and it is here that we get our first indication of discrepancy between theory and experiment. As noted above, the experimental peak frequency decreases with radius as expected (open symbols), but the decrease does not match that predicted by theory (filled symbols). Theory also predicts an increase of peak frequency with axial mode number n. We observe an increase, but it is not in accord with the theory.

We have also compared the amplitude of the experimentally measured flux resonances with the prediction of plateau regime theory. The result is shown in Figure 6. Although the curves are similar, several discrepancies are clear. As noted above the peaks (in this case the minima) of the resonances occur at slightly different frequencies. More importantly, the value of the experimental flux at the peak is roughly forty times smaller than the theoretical prediction. Lastly, although the theoretical curve passes smoothly through $\omega = 0$ with a significant positive flux, the experimental curve shows anomalously low transport near $\omega = 0$.

CONCLUSION

We have begun to test the resonant particle theory of asymmetry-induced transport under very simple conditions. Our initial results support the idea that reso-

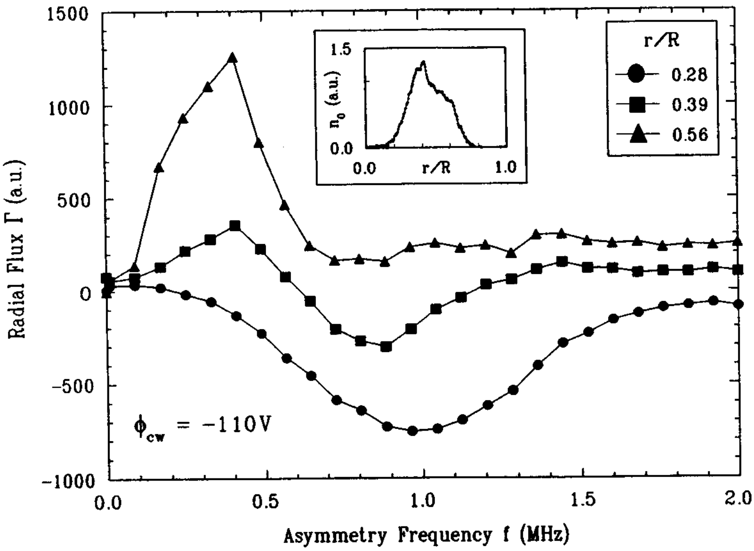

FIGURE 4. Radial particle flux at three radii as a function of asymmetry frequency for center wire bias $\phi_{cw} = -110V$. The shape of the flux curves is qualitatively consistent with that expected from theory.

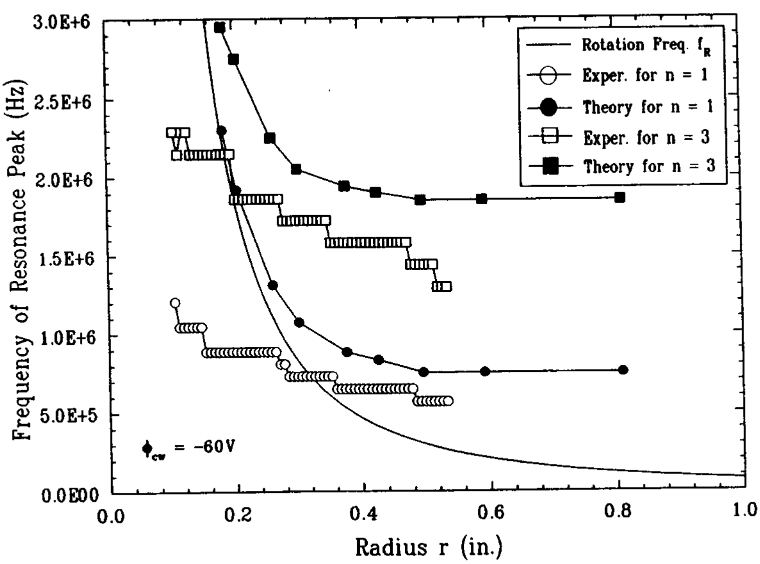

FIGURE 5. Variation of the flux resonance peak frequency with radius and axial mode number n. The open symbols give the experimental values and the corresponding closed symbols give the theory. For reference, the solid line gives the $E \times B$ rotation frequency f_R.

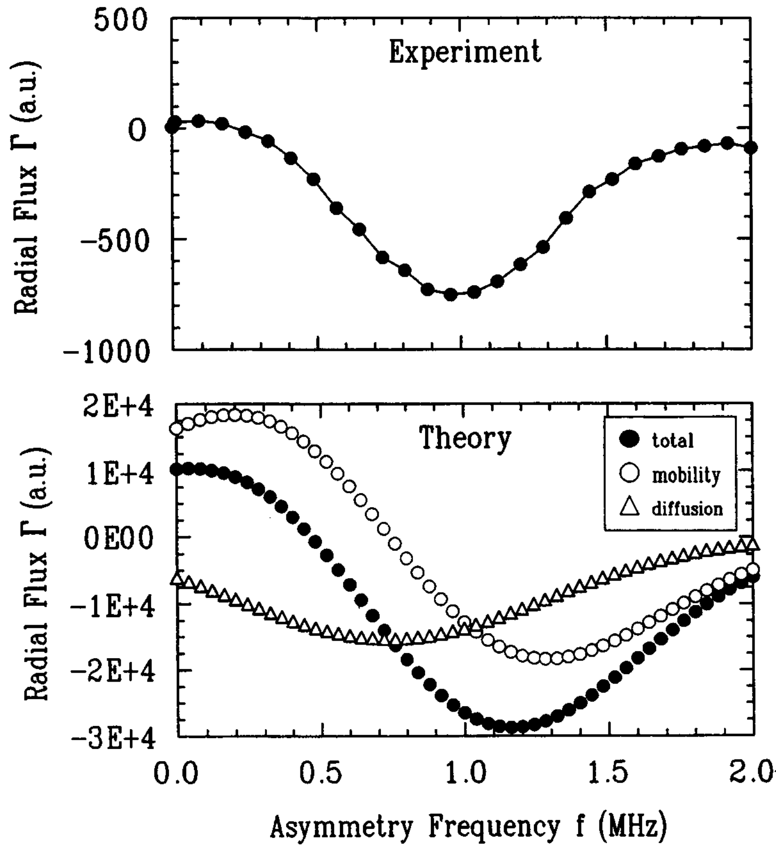

FIGURE 6. Absolute comparison between experimental flux and the prediction of plateau regime theory for center wire bias $\phi_{cw} = -110V$ and $r/R = 0.28$. The experimental flux is roughly forty times smaller than the theory predicts. The open symbols on the theory plot show the contributions of the diffusive and mobility-like terms in equation (4).

nant particles dominate the transport and we observe an amplitude scaling consistent with plateau regime theory. However, several discrepancies between theory and experiment are observed and it already seems clear that current theory does not give a complete description of this transport.

ACKNOWLEDGEMENTS

This work was supported by U.S. Department of Energy grant DE-FG03-98ER54457.

REFERENCES

1. J. S. DeGrassie and J. H. Malmberg, Phys. Fluids **23**, 63 (1980).
2. M. H. Douglas and T. M. O'Neil, Phys. Fluids **21**, 920 (1978).
3. J. H. Malmberg and C. F. Driscoll, Phys. Rev. Lett. **44**, 654 (1980).
4. C. F. Driscoll and J. H. Malmberg, Phys. Rev. Lett. **50**, 167 (1983).
5. C. F. Driscoll, K. S. Fine, and J. H. Malmberg, Phys. Fluids **29**, 2015 (1986).
6. D. L. Eggleston, T. M. O'Neil, and J. H. Malmberg, Phys. Rev. Lett. **53**, 982 (1984).
7. J. Notte and J. Fajans, Phys. Plasmas **1**, 1123 (1994).
8. X.-P. Huang *et al.*, Phys. Rev. Lett. **78**, 875 (1997).
9. D. Ryutov and G. Stupakov, JETP Lett. **26**, 174 (1978).
10. D. Ryutov and G. Stupakov, Sov. J. Plasma Phys. **4**, 278 (1978).
11. D. Ryutov and G. Stupakov, Sov. Phys. Dokl. **23**, 412 (1978).
12. R. Cohen, Comments Plasma Phys. Cont. Fusion **4**, 157 (1979).
13. R. Cohen, Nuclear Fusion **19**, 1579 (1979).
14. D. L. Eggleston and T. M. O'Neil, Phys. Plasmas **6**, 2699 (1999).
15. D. L. Eggleston and J. H. Malmberg, Phys. Rev. Lett. **59**, 1675 (1987).
16. D. L. Eggleston, Phys. Plasmas **4**, 1196 (1997).
17. D. L. Eggleston, Phys. Plasmas **1**, 3850 (1994).