Using variable-frequency asymmetries to probe the magnetic field dependence of radial transport in a Malmberg-Penning trap

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Abstract. A new experimental technique is used to study the dependence of asymmetry-induced radial particle flux Γ on axial magnetic field B in a modified Malmberg-Penning trap. This dependence is complicated by the fact that B enters the physics in at least two places: in the asymmetry-induced first order radial drift velocity $v_r = E_\theta/B$ and in the zeroth order azimuthal drift velocity $v_\theta = E_r/B$. To separate these, we employ the hypothesis that the latter always enters the physics in the combination $\omega - l\omega_R$, where $\omega_R = v_\theta/r$ is the column rotation frequency and ω and l are the asymmetry frequency and azimuthal mode number, respectively. Points where $\omega - l\omega_R = 0$ are then selected from a Γ vs r vs ω data set, thus insuring that any function of this combination is constant. When the selected flux Γ_{sel} is plotted versus the density gradient, a roughly linear dependence is observed, showing that this selected flux is diffusive. This linear dependence is roughly independent of the bias of the center wire in our trap ϕ_{cw}. Since in our experiment ω_R is proportional to ϕ_{cw}, this latter point shows that our technique has successfully removed any dependence on ω_R and its derivatives, thus confirming our hypothesis. The slope of a least-squares fitted line through the Γ_{sel} vs density gradient data then gives the diffusion coefficient D_0 under the condition $\omega - l\omega_R = 0$. Varying the magnetic field, we find D_0 is proportional to $B^{-1.33\pm0.05}$, a scaling that does not match any theory we know. These findings are then used to constrain the form of the empirical flux equation. It may be possible to extend this technique to give the functional dependence of the flux on $\omega - l\omega_R$.

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INTRODUCTION

The Malmberg-Penning non-neutral plasma trap continues to be of interest both as a platform for basic plasma physics studies and for its applications in charged particle storage and manipulation. While it is well established that electric and magnetic fields that break the cylindrical symmetry of these traps produce radial transport, a full understanding of the transport remains elusive. Indeed, our work, which focuses on the transport produced by applied electric asymmetries with frequency ω and axial and azimuthal wavenumbers n and l, has revealed serious discrepancies between experiment[1] and some of the predictions of theory[2].

Faced with these discrepancies, we have turned to developing an empirical model of the transport with an eye toward providing guidance for further theoretical development. A basic issue in this program is determining the magnetic field dependence of the transport. Although this scaling has been studied before[4, 5], there is no consensus of results. This may be due to the fact that the magnetic field B enters the transport physics in at least two ways. Firstly, in the zeroth order azimuthal $E \times B$ drift produced by the radial electric field $v_\theta = E_r/B$. This causes the particle guiding centers to drift around the trap axis with angular frequency $\omega_R = v_\theta/r$. Secondly, the magnetic field enters in the first order radial $E\times B$ drift produced by the applied asymmetry $v_r = E_\theta/B$. It is this drift which is responsible for the radial transport of particles. This dual dependence on magnetic field can be seen, for example, in the expression for the flux Γ from the plateau regime of resonant particle transport theory[2].

In this paper we apply a new experimental technique to remove the ω_R dependence and thus isolate any remaining magnetic field dependence. The technique is based on the hypothesis that the asymmetry frequency ω and ω_R always enter the transport physics in the combination $\omega - l\omega_R$. We then select from a Γ vs r vs ω data set those points where $\omega - l\omega_R = 0$, thus insuring that any function of this combination is constant. When the selected flux Γ_{sel} is plotted versus the density gradient ∇n_0, a roughly linear dependence is observed, showing that this selected flux is at least partially diffusive. This linear dependence is roughly independent of the center wire bias ϕ_{cw}. Since in our experiment $\omega_R \propto \phi_{cw}$, this latter point shows that our technique has successfully removed any dependence on ω_R and its derivatives, thus confirming our hypothesis. The slope of a least-squares fitted line through the Γ_{sel} vs ∇n_0 data then gives the diffusion coefficient under the condition $\omega - l\omega_R = 0$ which we call D_0. Varying the magnetic field, we find $D_0 \propto B^{-1.33\pm0.05}$. We then use these findings to constrain the form of the empirical flux equation[3].

EXPERIMENTAL DEVICE

Our transport studies are performed in the modified Malmberg-Penning trap shown in shown in Fig. 1. As in the standard trap design, a uniform axial magnetic field provides radial confinement of injected electrons, while negatively biased end cylinders (the injection gate and dump gate) provide axial confinement. Our device also operates in the standard inject-hold-dump cycle. A cycle begins by grounding the injection gate which allows electrons from the gun to flow into the central region. This injection gate is then returned to a negative bias which traps the electrons. After a chosen period of time, the dump gate is grounded and the electrons leave the trap and hit a positively biased phosphor screen. Analysis of the images on this screen provides the primary diagnostic.

The principal modification in our device is replacing the usual plasma column with a biased wire running along the axis of the trap. The wire provides a radial electric field to replace the field normally produced by the plasma column and allows the injected low density electrons to have the same zeroth-order dynamical motions (axial bounce and azimuthal $E\times B$ drift motions) as in a standard trap. The lower density (10^5 cm^{-3}) and high temperature (4 eV) of the electrons give a Debye length larger than the trap radius. Under these conditions, potentials in the plasma are essentially the vacuum potentials and previously encountered[6] complications due to collective effects are minimized[1]. Our design also allows the drift rotation frequency $\omega_R(r)$ to be easily adjusted by varying the center wire bias ϕ_{cw} since $\omega_R = \frac{-\phi_{cw}}{r^2 B \ln(R/a)}$ where R and a are the radii of the wall and the center wire, respectively. Despite these changes, the unperturbed confinement time has similar magnitude and shows[7] the same $(L/B)^2$ scaling found in higher

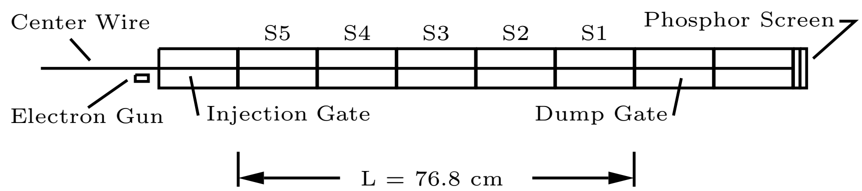

FIGURE 1. Schematic of the Occidental College Trap. The usual plasma column is replaced by a biased wire to produce the basic dynamical motions in low density electrons injected from an off-axis gun. The low density and high temperature of the injected electrons largely eliminate collective modifications of the vacuum asymmetry potential. The five cylinders (labeled S1 through S5) are divided azimuthally into eight sectors each.

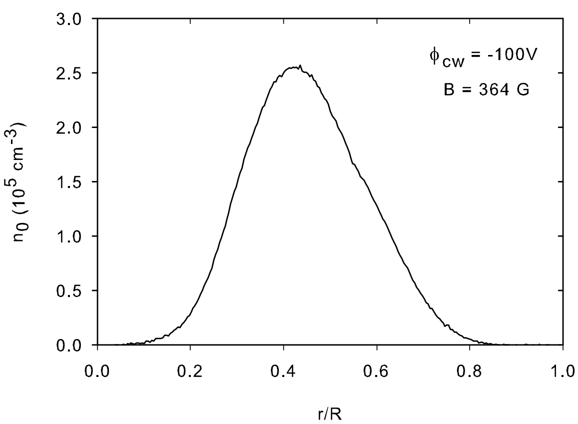

FIGURE 2. A typical density profile taken 1600 ms after injection.

density experiments, thus supporting the idea that the radial transport is primarily a single particle process and confirming the relevance of our experiments to standard trap physics.

A unique feature of our device is that the entire confinement region is sectored (five cylinders, labeled S1 through S5 in Fig. 1, with eight azimuthal divisions each). This allows us to apply a simple, known asymmetry by selecting the amplitude and phase of the voltages applied to each sector to produce a helical standing wave of the form $\phi(r,\theta,z,t) = \phi_W \left(\frac{r}{R}\right)^l \cos\left(\frac{n\pi z}{L}\right)\cos(l\theta - \omega t)$ where ϕ_W is the asymmetry potential at the wall (typically 0.2 V), R is the wall radius (3.82 cm), L is the length of the confinement region (76.8 cm), n and l are the axial and azimuthal Fourier mode numbers, respectively, and z is measured from one end of the confinement region. For these experiments $n = l = 1$ and the relative phases of the applied voltages are adjusted so that the asymmetry rotates in the same direction as the zeroth-order azimuthal $E \times B$ drift.

Data acquisition for these transport studies can be summarized as follows; details have been given elsewhere [1, 8]. Electrons injected into the trap from an off-axis gun are quickly dispersed into an annular distribution. At a chosen time (here, 1600 ms after injection), the asymmetries are switched on for a period of time δt (here, 100 ms) and then switched off. At the end of the experiment cycle, the electrons are dumped axially onto a phosphor screen and the resulting image is digitized using a 512×512 pixel charge-coupled device camera. A radial cut through this image gives the density profile $n_0(r)$ of the electrons. A typical profile is shown in Fig. 2. Shot-to-shot variation in the number of injected electrons is less than 1% and the data is very reproducible. Calibration is provided by a measurement of the total charge being dumped. Profiles are taken both with the asymmetry on and off, and the resulting change in density $\delta n_0(r)$ is obtained. The background transport is typically small compared to the induced transport and is subtracted off. If the asymmetry amplitude is small enough and the asymmetry pulse length δt short enough, then $\delta n_0(r)$ will increase linearly in time [8]. We may then approximate $dn_0/dt \simeq \delta n_0(r)/\delta t$ and calculate the radial particle flux $\Gamma(r)$ (assuming $\Gamma(r=a)=0$):

$$\Gamma(r) = -\frac{1}{r}\int_a^r r' dr' \cdot \frac{dn_0}{dt}(r') \tag{1}$$

Here a is the radius of the central wire (0.178 mm). The entire experiment is then repeated for a series of asymmetry frequencies ω and the resulting flux versus radius and frequency data saved for analysis.

EXPERIMENTAL RESULTS

It is easy to show experimentally that the transport depends separately on both ω and ω_R and that the form of the transport equation is more complicated than a simple Fick's Law dependence $\Gamma = -D\nabla n_0$. Some typical data is shown in Fig. 3. In Fig. 3a we plot the radial particle flux Γ versus radius r for three representative asymmetry frequencies to illustrate the dependence on ω. In Fig. 3b the same data is plotted versus density gradient ∇n_0 to show that there is no simple relationship between Γ and ∇n_0. Similar plots holding ω constant and varying ϕ_{cw} (and thus ω_R) demonstrate the dependence of the flux on ω_R[3].

We now apply the hypothesis that ω and ω_R always enter the transport physics in the combination $\omega - l\omega_R$. We take Γ vs r data for a number (typically 26) of asymmetry frequencies ω. Since $l=1$ in our experiments, these frequencies are chosen to be within the range of ω_R values, i.e., $\omega_R(R) < \omega < \omega_R(a)$. We then select from this Γ vs r vs ω dataset those points where $\omega - l\omega_R = 0$, thus insuring that any function of this combination is constant. We do this as follows: for each experimental value of ω, we determine the radial position r_{sel} where $\omega - l\omega_R = 0$, interpolating between data points if necessary. We then take from the Γ vs r data for that ω the single flux value Γ_{sel} that occurs at r_{sel}. After this is repeated for each ω, we have Γ_{sel} vs r_{sel} with r_{sel} spanning the range of radius values. Since the plasma parameters are independent of ω, ∇n_0 does not change with ω and we can also form Γ_{sel} vs ∇n_0.

When the selected flux is plotted versus the density gradient ∇n_0, a roughly linear dependence is observed and this dependence is roughly independent of the center wire bias ϕ_{cw}. Typical data is shown in Fig. 4a. The linearity of the plot shows that the selected flux has the form $\Gamma_{sel} = m\nabla n_0 + \Gamma_0$, where m and Γ_0 are constants. In particular, m and Γ_0 are not functions of ω or ω_R. The first follows from the fact that the data points in Fig. 4a are all at different frequencies and the second follows from the lack of dependence on

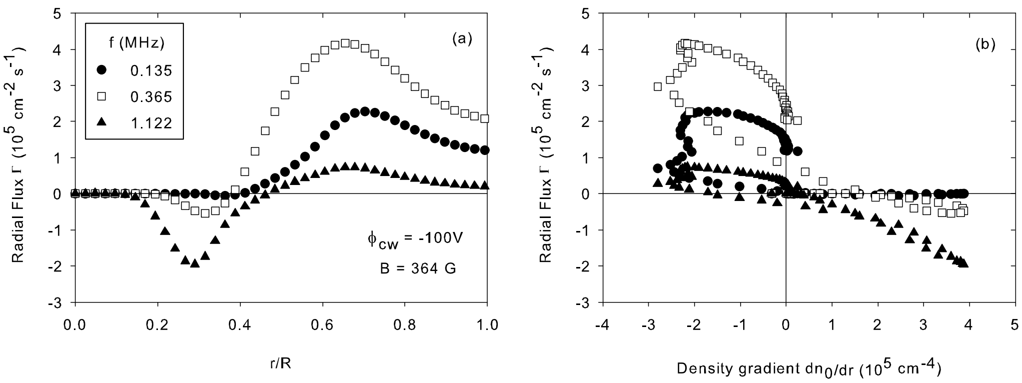

FIGURE 3. (a) Plot of typical flux versus scaled radius data for three representative asymmetry frequencies. (b) Plot of the same flux data versus density gradient. The number of plotted points has been adjusted for clarity. The plots show that the flux depends on the asymmetry frequency and does not follow a simple Fick's Law dependence on density gradient.

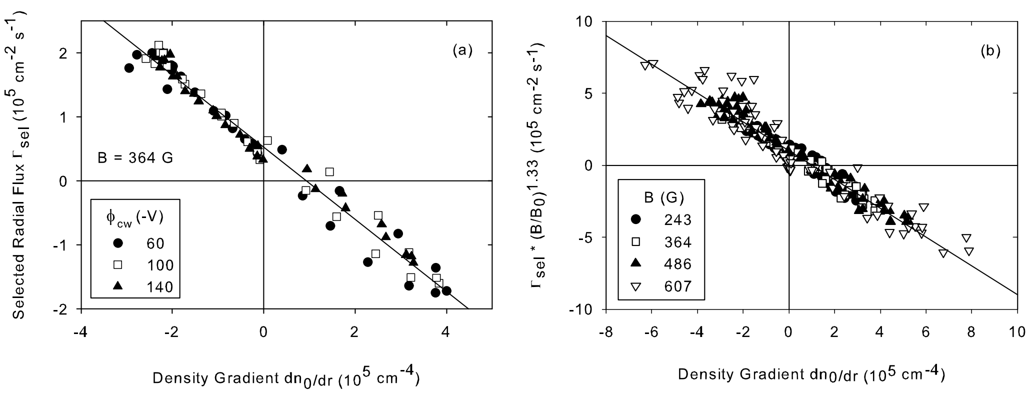

FIGURE 4. a) Selected flux versus density gradient with center wire bias as a parameter. The slope of a fitted line gives the diffusion coefficient. b) A universal curve results when the selected flux data from three center wire biases for each of four magnetic fields is multiplied by a scaling factor $(B/B_0)^{1.33}$ and plotted versus the density gradient dn_0/dr.

ϕ_{cw}. Since we know that, in general, the flux depends separately on both ω and ω_R, the independence of Γ_{sel} on these quantities supports our hypothesis that they enter the physics only in the combination $\omega - l\omega_R$. We also note that, since the points in Fig. 4a come from different radii, m and Γ_0 are not strong functions of r either, although the deviations from linearity may indicate a weak dependence on r.

Finally, the slope of a least-squares fitted line to the plot in Fig. 4a then gives the quantity m. For four values of magnetic field spanning 243-607 G, we find[3] $m \propto B^{-1.33\pm0.05}$. A similar procedure using the y-intercept of the fitted lines gives $\Gamma_0 \propto B^{-1.13\pm0.10}$.

DISCUSSION

The magnetic field scalings for m and Γ_0 are similar enough to consider a common scaling for both. This is of interest for comparison with the common theoretical form for the flux. In Fig. 4b we apply a scaling of $B^{1.33}$ to all of our data (three center wire biases for each of four magnetic fields) and obtain a universal curve of the form $(B/B_0)^{1.33}\Gamma_{sel} = -D_0(\nabla n_0 + f_0)$, where $B_0 = 233$ G is a conveniently selected constant. A least-squares fit to the scaled data gives $D_0 = 1.00\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ and $f_0 = -1.01 \times 10^5\ \mathrm{cm}^{-4}$. This magnetic field scaling does not match the theoretical B^{-2} plateau regime scaling or the more complicated B-scaling of the banana regime[2], or any other theoretical scaling of which we are aware.

Of course, our universal curve only gives the flux for points where $\omega - l\omega_R = 0$. It does, however, allow us to say something about the form of the general flux equation. Our data tell us that the general flux must be a function of $\omega - l\omega_R$ and that the flux equation must reduce to the equation for Γ_{sel} when $\omega - l\omega_R = 0$. Without further information, we must thus allow both D_0 and f_0 to become functions of $\omega - l\omega_R$:

$$\Gamma = -(B_0/B)^{1.33} D(\omega - l\omega_R)[\nabla n_0 + f(\omega - l\omega_R)] \tag{2}$$

where $D(\omega - l\omega_R = 0) \equiv D_0$ and $f(\omega - l\omega_R = 0) \equiv f_0$.

CONCLUSION

We have applied a new experimental technique to study the magnetic field dependence of asymmetry-induced transport in a modified Malmberg-Penning trap. The technique allows us to remove the ω_R-dependence from our data and thus isolate the remaining magnetic field dependence. The technique works reasonably well and gives a diffusion coefficient that scales like $B^{1.33}$. This scaling does not match that of any known theory.

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REFERENCES

1. D.L. Eggleston and B. Carrillo, Phys. Plasmas **10**, 1308 (2003).
2. D. L. Eggleston and T. M. O'Neil, Phys. Plasmas **6**, 2699 (1999).
3. D.L. Eggleston and J.M. Williams, Phys. Plasmas **15**, 032305 (2008).
4. J. Notte and J. Fajans, Phys. Plasmas **1**, 1123 (1994).
5. J. M. Kriesel and C. F. Driscoll, Phys. Rev. Lett. **85**, 2510 (2000).
6. D. L. Eggleston, T. M. O'Neil, and J. H. Malmberg, Phys. Rev. Lett. **53**, 982 (1984).
7. D.L. Eggleston, Phys. Plasmas **4**, 1196 (1997).
8. D. L. Eggleston and B. Carrillo, Phys. Plasmas **9**, 786 (2002).