Tumor Growth
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What is a Tumor?

Tumors usually start as a mild disorder of cell behavior that slowly develops through well characterized stages 0-IV into full cancer. The cancer is not fully developed until long after the initial mutation has occurred. Eg. leukemia rates in Hiroshima, Japan did not spike until 5 years after the drop of the H-bomb. Pre-cancer cells, or stage 0 cancer, take nutrients from the host's cells by diffusion which provides nutrients only to cells on the surface of the tumor. Active cancer - stage I and onward - harnesses the host's blood supply to deliver nutrients and oxygen to the innermost cells of the tumor. After securing the host organism's blood supply, the tumor can grow 16,000 times its volume in a matter of weeks!

Concentration of Oxygen in the Tumor

The innermost cells of the tumor are the least likely to receive nutrients like oxygen. When the innermost tumor cells fail to get enough oxygen, they die. The dead cell mass left behind is called the necrotic core. The following equation models the concentration of oxygen in the tumor, which is 0 in the necrotic core, where the radius is small:

\[ c(r) = \frac{k r^2}{6D} \cdot \frac{A}{r} + B \]

Fighting the Tumor

When the body's immune system is able to recognize a tumor, it launches a cytotoxic response producing effector cells that attach to tumor cells and destroy them both. A tumor growing unattacked by the immune system is modeled by the first equation. The body's immune response can be modeled by the second and third equations:

\[ \frac{\partial X}{\partial r} \left( 1 - \frac{X}{K} \right) + D \nabla^2 X \]
\[ \frac{\partial E}{\partial r} = -k_1 EX + k_2 C \]
\[ \frac{\partial C}{\partial r} = k_1 EX \]

Combining the equations for tumor growth and the immune response, a model of the fight:

\[ \frac{\partial X}{\partial r} \left( 1 - \frac{X}{K} \right) = \frac{k_3 E_0 X}{k_2 + k_1 X} + D \nabla^2 X \]

Tumor Growth in the Early Stages

The logistic equation models the tumor size and rate of growth during the early stages of growth. The rate of tumor growth, \( \frac{dN}{dt} \), depends on the size of the tumor, \( N \). The larger the tumor, the more rapid the tumor growth will be. Let \( r \) be a rate constant and \( K \) is the final size of the tumor.

\[ \frac{dN}{dt} = rN - \left( 1 - \frac{N}{K} \right) \]

This equation can be generalized into the Von Bertalanffy equation on the left. The Gompertz Equation (right) can also be used to model early tumor growth:

\[ \frac{dN}{dt} = aN^b - bN^c \]
\[ \frac{dN}{dt} = -bN \log \frac{N}{K} \]

References