1. (#5.6.1 in Strauss)

   (a) Solve as a series the equation $u_t = u_{xx}$ in $(0, 1)$ with $u_x(0, t) = 0$, $u(1, t) = 1$, and $u(x, 0) = x^2$. Compute the first two coefficients explicitly.

   (b) What is the equilibrium state (the term that does not tend to zero)?

2. (#5.6.5 in Strauss) Solve $u_{tt} = c^2 u_{xx} + e^t \sin 5x$ for $0 < x < \pi$, with $u(0, t) = u(\pi, t) = 0$ and the initial conditions $u(x, 0) = 0$, $u_t(x, 0) = \sin 3x$.

3. (#5.6.8 in Strauss) Solve $u_t = ku_{xx}$ in $(0, l)$, with $u(0, t) = 0$, $u(l, t) = At$, $u(x, 0) = 0$, where $A$ is a constant.

4. (#5.6.9 in Strauss) Use the method of subtraction to solve $u_{tt} = 9u_{xx}$ for $0 \leq x \leq 1 = l$, with $u(0, t) = h$, $u(1, t) = k$, where $h$ and $k$ are given constants, and $u(x, 0) = 0$, $u_t(x, 0) = 0$.

5. Consider the Dirichlet problem for the wave equation with periodic forcing

   \[
   \begin{cases}
   u_{tt} - c^2 u_{xx} = f(x) \cos \omega t, & \text{for } 0 < x < \pi, \\
   u(x, 0) = \phi(x), & u_t(x, 0) = \psi(x), \\
   u(0, t) = u(\pi, t) = 0.
   \end{cases}
   \]

   Solve the problem in the series form, if $\omega \neq cm$ for any integer $m$. (Hint: Use the method of undetermined coefficients for the ODE satisfied by the Fourier coefficients of the solution.)

6. Solve the previous problem with $\omega = cm$ for some integer $m$, and show that one encounters resonance in this case.

7. Consider the \textit{damped} wave equation with periodic forcing, and the corresponding Dirichlet problem

   \[
   \begin{cases}
   u_{tt} - c^2 u_{xx} + ru_t = f(x) \cos \omega t, & \text{for } 0 < x < \pi, \\
   u(x, 0) = \phi(x), & u_t(x, 0) = \psi(x), \\
   u(0, t) = u(\pi, t) = 0.
   \end{cases}
   \]

   (a) Solve the problem if $r$ is small ($0 < r < 2c$), and show that the damping prevents resonance from occurring.

   (b) Show that no matter what the initial data $(\phi, \psi)$ are, the solution to the above problem always converges to an asymptotic solution $U(x, t)$ as $t \to \infty$, and find this asymptotic solution.