1. (#2.1.1 in Strauss) Solve $u_{tt} = c^2 u_{xx}$, $u(x,0) = e^x$, $u_t(x,0) = \sin x$.

2. (#2.1.5 in Strauss) Let $\phi(x) \equiv 0$ and $\psi(x) = 1$ for $|x| < a$ and $\psi(x) = 0$ for $|x| \geq a$. Sketch the string profile ($u$ versus $x$) at each of the successive instants $t = a/2c, a/c, 3a/2c, 2a/c$, and $5a/c$. [Hint: Calculate

$$u(x,t) = \frac{1}{2c} \int_{x-ct}^{x+ct} \psi(s) \, ds = \frac{1}{2c} \{\text{length of } (x - ct, x + ct) \cap (-a,a)\}.$$  

Then $u(x,a/2c) = (1/2c)\{\text{length of } (x-a/2, x+a/2) \cap (-a,a)\}$. This takes on different values for $|x| < a/2$, for $a/2 < x < 3a/2$, and for $x > 3a/2$. Continue in this manner for each case.]

3. (#2.1.7 in Strauss) If both $\phi$ and $\psi$ are odd functions of $x$, show that the solution $u(x,t)$ of the wave equation is also odd in $x$ for all $t$.

4. (#2.1.9 in Strauss) Solve $u_{xx} - 3u_{xt} - 4u_{tt} = 0$, $u(x,0) = x^2$, $u_t(x,0) = e^x$. (Hint: Factor the operator into first order operators, similar to what we did for the wave equation.)

5. (#2.2.3 in Strauss) Show that the wave equation has the following invariance properties.

   (a) Any translate $u(x - y, t)$, where $y$ is fixed, is also a solution.

   (b) Any derivative, say $u_x$, of a solution is also a solution.

   (c) The dilated function $u(ax, at)$ is also a solution, for any constant $a$.

6. (#2.2.4 in Strauss) If $u(x,t)$ satisfies the wave equation $u_{tt} = u_{xx}$, prove the identity

$$u(x + h, t + k) + u(x - h, t - k) = u(x + k, t + h) + u(x - k, t - h)$$

for all $x, t, h$ and $k$. Sketch the quadrilateral $Q$ whose vertices are the arguments in the identity.

7. (#2.2.5 in Strauss) For the damped string, $u_{tt} - c^2 u_{xx} + ru_t = 0$, $r > 0$, show that the energy decreases.