Lines

A **Line** in $\mathbb{R}^2$ is uniquely determined by

- A point $P$ on the line *and* a vector normal to the line, $\mathbf{n}$
- A point $P$ on the line *and* a vector parallel to the line, $\mathbf{d}$.
- Two points on the line, $P$ and $Q$.

Let $\mathbf{p}$ be the position vector of the point $P = (x_0, y_0)$, then for the point with position vector $\mathbf{x} = (x, y)$ to be on the line, the vector $\mathbf{x} - \mathbf{p}$ must be on the line as well. This can be written as

\[
(\mathbf{x} - \mathbf{p}) \cdot \mathbf{n} = 0 \quad \text{(normal form, only in } \mathbb{R}^2) \\
(\mathbf{x} - \mathbf{p}) = t\mathbf{d} \quad \text{(vector form, works in any } \mathbb{R}^n) \\
(\mathbf{x} - \mathbf{p}) = t(\mathbf{q} - \mathbf{p})
\]

The third equation is a particular case of the vector form, since the vector $\mathbf{q} - \mathbf{p}$ is parallel to the line. The vector form of the equation works in any higher dimensions. If we write the vector form in terms of components, this will give the **parametric equations** of the line.

\[
\begin{aligned}
    x &= x_0 + ta \\
y &= y_0 + tb
\end{aligned}
\]
Exercises:

1. Find the vector and parametric equations of the line passing through $p = (4, -1, 3)$ and $Q = (2, 1, 3)$.

2. Find the vector form of the equation of the line in $\mathbb{R}^3$ that passes through $P = (-1, 0, 3)$ and is parallel to the line with parametric equations

$$\begin{align*}
\begin{cases}
x = 1 - t \\
y = 2 + 3t \\
z = -2 - t
\end{cases}
\end{align*}$$