Practice First Midterm Exam

Please note that the practice exam is meant to illustrate the format and the difficulty level of the actual exam. The types of problems and the represented topics on the actual exam may differ from those on the practice exam.

No calculators, closed notes and books!
Please write neatly and provide ample explanations. Answers without proper justification will not be counted. Check your work carefully. Cross out anything you don’t want to be graded.

1. [10] Determine whether the following sequences converge or diverge. Find the limits of convergent sequences.
   
   (a) \( \left\{ \frac{e^n + e^{-n}}{e^{2n} + 1} \right\} \)

   (b) \( \{n^2 e^{-n}\} \)

   (c) \( \{\ln(n+1) - \ln n\} \)

   (d) \( \left\{ \frac{n}{1 + \sqrt{n}} \right\} \).

2. [10] Determine whether the following series converge or diverge.

   (a) \( \sum_{k=1}^{\infty} \frac{3^k + 2^k}{6^{k-1}} \)

   (b) \( \sum_{k=1}^{\infty} ke^{-k^2} \)

   (c) \( \sum_{k=1}^{\infty} \frac{\sqrt{3}}{k} \)

   (d) \( \sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3 + 1}} \).

3. [10] For what values of \( p \) does the series

   \[ \sum_{k=2}^{\infty} \frac{1}{k^p \ln k} \]

   converge?

4. [10] Show that if \( a_k \geq 0 \) and \( \sum_{k=1}^{\infty} a_k \) converges, then so does \( \sum_{k=1}^{\infty} a_k^2 \).

5. [10]

   (a) Does the series \( \sum_{k=1}^{\infty} \) defined by the equations

   \[ a_1 = 1, \quad a_{k+1} = \frac{2 + \sin k}{\sqrt{k}} a_k. \]

   converge or diverge? Explain

   (b) What is the \( n^{th} \) order Maclaurin polynomial of \( xe^x \)?