
Mathematical Modeling

Math 396 Fall 2008

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Fowler 110 Thu 1:30- 2:55pm

<http://faculty.oxy.edu/ron/math/396/08/>

Week 7: Thursday October 9

TITLE Monte Carlo Simulation

CURRENT READING Meerschaert, Chapter 9 and Giordano, Chapter 5

SUMMARY

This week we will continue our look at discrete stochastic modelling by being introduced to Monte Carlo Simulation.

DEFINITION: Monte Carlo Simulation

Monte Carlo simulation chain is a general modelling technique for simulating random behavior. It is a stochastic (or probabilistic) modelling approach. Generally the simulation is done with the aid of a computer.

EXAMPLE 5.1 (Giordano)

Using a probabilistic technique to model deterministic behavior: area under a curve.

Consider the following pseudo-code algorithm for estimating the area $\int_a^b f(x) dx$

INPUT: Total Number of random points to be generated in the simulation = N

OUTPUT: AREA = approximate area under the specified curve $y = f(x)$ over the interval $a \leq x \leq b$ where $0 \leq f(x) \leq M$

BEGIN

STEP 1: Initialize COUNTER = 0

STEP 2: FOR $i = 1, 2, \dots, n$ DO steps 3-5

 STEP 3: Calculate random coordinates x and y such that x lies in $[a, b]$
 and y lies in $[0, M]$

 STEP 4: Calculate $f(x)$ for the random x coordinate

 STEP 5: IF $y < f(x)$, THEN increment the COUNTER by 1.
 Otherwise, leave COUNTER as is.

STEP 6: Calculate AREA = $M \cdot (b-a) \cdot \text{COUNTER} / N$

STEP 7: OUTPUT (AREA)

END

Using Monte Carlo Simulation To Estimate Area

Let's try and use the algorithm to estimate the value of $\int_0^2 x^3 dx$. Record your estimates in the Table Below. Use the Excel Spreadsheet `Area.xls` in `Start--> My Computer-->S ('stuserver')--> Math Courses--> Math 396`

N	AREA

Using Monte Carlo Simulation To Simulate Probabilistic Behavior

EXAMPLE 9.1 (Meerschaert)

Arriving on your vacation you are dismayed to learn that the local weath service forecats a 50% chance of rain every dayy this week. What are the chances of three consecutive rainy days?

We'll use the Matlab files `rainy.m` in `Start--> My Computer-->S ('stuserver')--> Math Courses--> Math 396`

We'll run the program numerous time (simulating the weather) and then record the number of rainy weeks occur out of a 100 simulated weeks. Each of you should run the program 10 times or so and record your results.

RUN	NUMBER OF RAINY WEEKS

```

% ALGORITHM TO SIMILUATE RAINY WEEKS PROBLEM in Meerschaert
% Ron Buckmire, October 2008
%
% rainy.m
%
% p = probability of 1 rainy day
% n = number of weeks to simulate
% S = number of Rainy weeks
%
p = input('Input Rainy Day Probability\n');
n = input('Input Number of Times To Simulate\n');
S = 0; % initialize number of Rainy weeks
for nloops=1:n
    Y = rainyday(p); % Generate Random XY Point
    S = S+Y;
end
disp([' Number of Rainy Weeks = ' num2str(S)]);

```

```

% ALGORITHM TO SIMILUATE RAINY DAY PROBLEM in Meerschaert
function Y=rainyday(p)
% Ron Buckmire, October 2008
%
% rainyday.m
%
% p = probability of 1 rainy day
% X = 1 if RAINY on DAY t
% X = 0 if RAINY on day t
% Y = 1 if >= 3 rainy days consecutively
% Y = 0 otherwise
%
%p = input('Input Rainy Day Probability\n');
Counter = 0; % initialize counter
Y = 0; % initialize Y
for t=1:7
    Rrand=rand;
    if (Rrand < p)
        X=1;
    else
        X=0;
    end
    if (X==1)
        Counter= Counter +1;
    else
        Counter = 0;
    end
    if (Counter >= 3)
        Y=1;
    end
end
end

```

Homework Question (Week #7) for Math 396

Using Monte Carlo simulation, write (and implement!) an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter-circle

$$Q = \{(x, y) | x^2 + y^2 = 1, x \geq 0, y \geq 0\}$$

where the quarter-circle is taken to be inside the square

$$S = \{(x, y) | 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$$

HINT: Use the fact that $\frac{\pi}{4} = \frac{\text{area of } Q}{\text{area of } S}$