
Mathematical Modeling

Math 396 Fall 2008
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Fowler 110 Thu 1:30- 2:55pm
<http://faculty.oxy.edu/ron/math/396/08/>

Class 3: Thursday September 11

TITLE Modeling Using Optimization; Introduction to Sensitivity

CURRENT READING Meerschaert,

SUMMARY

We'll continue our review of modelling techniques by applying techniques from calculus: single-variable and multi-variable optimization.

“The Pig Problem” is taken from *Mathematical Modeling* (2nd Edition) by Mark M. Meerschaert published by Academic Press (in 1999). He popularizes the **5 Step Modeling Approach**:

1. Ask the question
2. Select the modeling approach
3. Formulate the model
4. Solve the model
5. Answer the question

The Pig Problem

A pig weighing 200 pounds gains 5 pounds per day and costs 45 cents a day to keep. The market price for pigs is 65 cents per pound but is falling 1 cent per day. When should the pig be sold?

Sensitivity Analysis

Suppose that one is uncertain about the exact value of a parameter in the problem. One can conduct what is known as a sensitivity analysis into how the model would change with respect to changes in parameter values.

DEFINITION: Sensitivity

Suppose a parameter r results in an answer x to a problem. The sensitivity of x with respect to r is denoted as $S(x, r)$ and is calculated as

$$S(x, r) = \lim_{\Delta r \rightarrow 0} \frac{\Delta x/x}{\Delta r/r} = \lim_{\Delta r \rightarrow 0} \frac{\Delta x}{\Delta r} \frac{r}{x} = \frac{dx}{dr} \cdot \frac{r}{x}$$

EXAMPLE

In the Pig Problem, what is the sensitivity of the time to sell x with respect to the rate r at which the price for pigs is falling?

Exercise

In the Pig Problem, what is the sensitivity of the time to sell x with respect to the rate g at which the weight of the pig is increasing per day?

The Pig Problem, Revisited

Reconsider the pig problem, but now take into account the fact that the growth rate of the pig is not constant. Assume that the pig is young, so that the growth rate is increasing, but is proportional to the pig's weight. When should the pig be sold for maximum profit?

(Your objective function for profit $f(x)$ versus time to sell x should be
 $f(x) = (0.65 - 0.01x)(200e^{0.025x}) - 0.45x$.)

Theorem

If y is directly proportional to z , then $S((x, y) = S(x, z)$ but if y is inversely proportional to z then $S(x, y) = -S(x, z)$.

Homework

HINT: You may have to numerically estimate these sensitivities.

(GROUP 1) What is the sensitivity of the time to sell with respect to the growth rate of the pig?

(GROUP 2) What is the sensitivity of the time to sell with respect to the initial weight of the pig?