History of Mathematics

Math 395 Spring 2010 Fowler 310 MWF 10:30am - 11:25am

http://faculty.oxy.edu/ron/math/395/10/

Class 20: **Friday March 24**

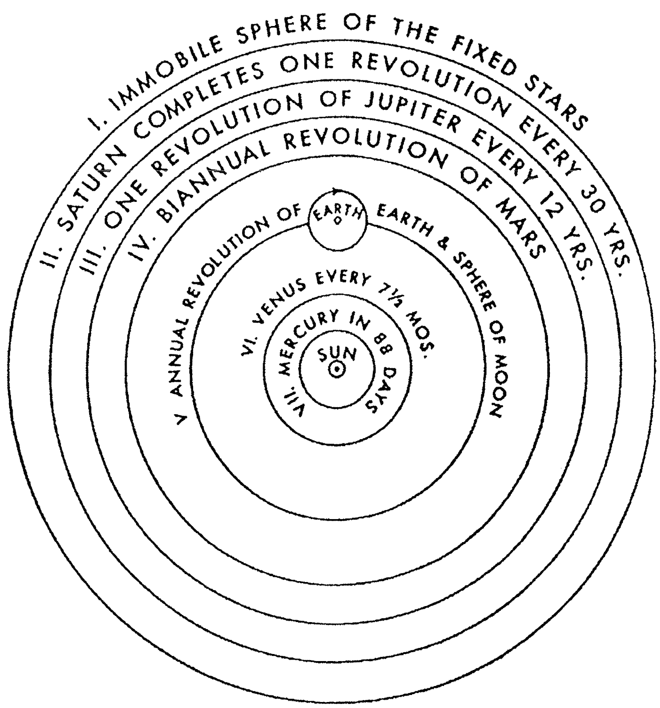
TITLE Mathematical Methods of the Renaissance Period
CURRENT READING: Katz, §13
NEXT READING: Katz, §14

Homework #8 DUE Friday April 2 (in class)
Katz, p. 418: #4, #8, #21, #30, #37. EXTRA CREDIT: page 419, #29.

SUMMARY
We will look at some of the mathematical techniques developed during the Renaissance.

Johannes Müller better known as **Regiomontanus** (1436-1476) wrote the first "pure" trigonometry Europe an text, *De Triangulis Omnimodis* (*On Triangles of Every Kind*) which became the most influential book on the development of astronomy and trigonometry in the 16th century after it was published more than 70 years after his death, in 1533.

Nicolaus Copernicus (1473-1543) developed a heliocentric model of the Universe in direct challenge to the model Ptolemy had published in *The Almagest*.



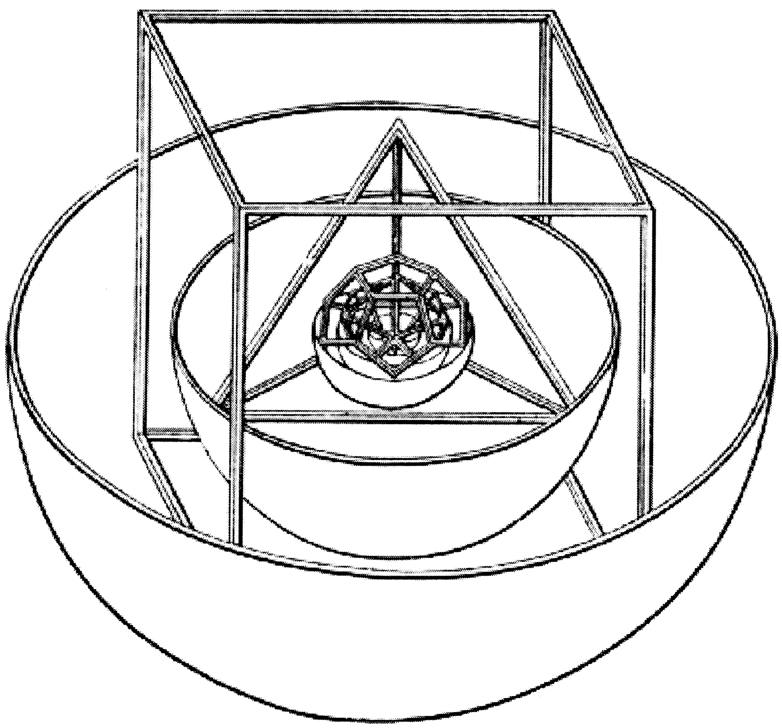
RECALL

Q: Was Copernicus the first person to propose a model of the Universe with the Sun at the Center?
A: No, the Greeks proposed such a model (**Aristarchus of Samos** (c. 310-230 BCE) as did **Aryhabata** of India. Copernicus was the first to propose such a model supported by calculations and observations.

Johannes Kepler (1571-1630) succeeded **Tycho Brahe** (1546-1601) as Imperial Mathematician of Emperor Rudolph II in Prague. Brahe was a Danish astronomer who made amazingly copious and accurate astronomical observations which Kepler was able to rely on in deriving Kepler's Laws.

In *Mysterium cosmographicum* (*The Secret of the Universe*), Kepler explained why there were exactly 6 known planets by relating them to the 5 Platonic solids. Between each two spheres in space containing the orbits of adjacent planets was one of these regular solids.

The order went: Saturn, **cube**, Jupiter, **tetrahedron**, Mars, **dodecahedron**, Earth, **icosahedron**, Venus, **octahedron**, Mercury. The idea was that the ratio of the sizes of the orbits was related to the similar ratio of the size of the spheres which circumscribed the regular solids. For example, the diameter of Jupiter's orbit is triple that of Mars, and the sphere circumscribing the tetrahedron is triple the sphere inscribed by the tertrahedron. The ratios didn't really work out for the other planets but Kepler was convinced that his model was correct regardless of data.

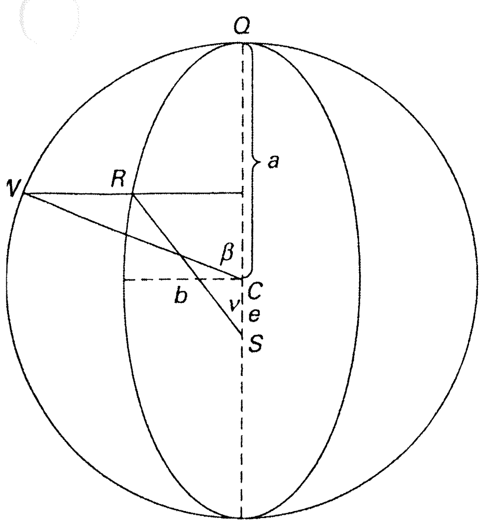


Kepler's Laws of Planetary Motion
The First Law: A planet travels in an ellipse around the sun with sun at one focus.

The Second Law: A planet sweeps out equal areas in equal times.

The Third Law: "It is absolutely certain and exact that the ratio which exists between the periodic times of any two planets is precisely the ratio of the $\frac{3}{2}$th power of the mean distances [of the planet to the sun]."

Kepler's Proof That Orbit of Mars Is Elliptical



Kepler finally discovered the correct result, that the distance given by $\rho = 1 + e\cos\beta$ should be laid off from the sun so that the endpoint is on a line perpendicular to the line CQ, where β is the angle that CQ makes with the line from C to the intersection W of that perpendicular with the auxiliary circle (Fig. 13.28). (One should note that the difference between Kepler's first idea and this one is extremely small, producing discrepancies of at most about 5′ of arc.) Kepler was able to demonstrate that the curve he now produced was an ellipse, using an argument summarized here in modern notation. Assume that an ellipse is centered at C with $a = 1$ and $b = 1 - \frac{e^2}{2}$, where $e = CS$. This ellipse can be thought of as being formed from the circle of radius 1 by reducing all the ordinates perpendicular to QC in the proportion b. If ν represents the angle at S subtended by the arc RQ, then $\rho\cos\nu = e + \cos\beta$ and $\rho\sin\nu = b\sin\beta$. Squaring the two equations and adding gives

$$\rho^2 = e^2 + 2e\cos\beta + \cos^2\beta + \left(1 - \frac{e^2}{2}\right)^2 \sin^2\beta$$

$$= e^2 + 2e\cos\beta + 1 - e^2\sin^2\beta + \frac{e^4}{4}\sin^2\beta.$$

Neglecting the term in e^4 then produces the result

$$\rho^2 = 1 + 2e\cos\beta + e^2\cos^2\beta = (1 + e\cos\beta)^2.$$

Thus, the equation of the ellipse can be written as $\rho = 1 + e\cos\beta$, exactly the same equatio as already derived for the curve of the orbit itself. In addition, the distance c of the center o the ellipse from the focus is given by

$$c^2 = 1 - b^2 = 1 - \left(1 - \frac{e^2}{2}\right)^2 = e^2,$$

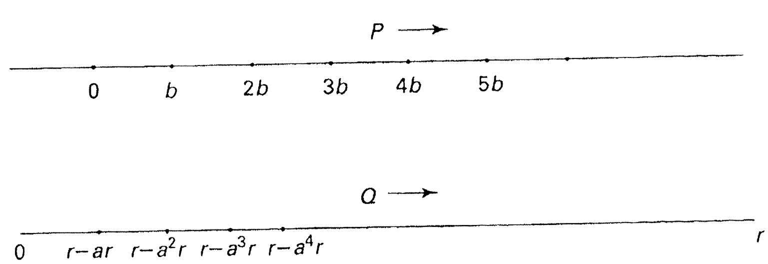
It follows then that the sun is at one focus of the ellipse and that e is the eccentricity of the ellipse.

Development of Logarithms

John Napier (1550-1617) in *Mirifici logarithmorim canonis description (Description of the Wonderful Canon of Logarithms)*, published in 1614 described the concept this way:

> **"the logarithm of a given sine is that number which has increased arithmetically with the same velocity as that with which radius began to decrease geometrically, and in the same time as radius has decreased to the given [number]."**

In other words, in the picture below, the point P is moving with constant (linear) velocity while Q is moving geometrically (exponentially) then when P has reached a distance y from the origin when Q has reached a distance x from the origin, we say $\boldsymbol{y}$ **is the logarithm of** $\boldsymbol{x}$.



Using differential equations, this corresponds to

$$\frac{dx}{dt} = -x, \quad x(0) = r; \qquad \frac{dy}{dt} = r, \quad y(0) = 0$$

Example

Solve the ODEs and obtain an expression for y=Nlog(x), y is the Napier logarithm of x. NOTE that the base r Napier used was r=10^7

GroupWork

Napier developed his logarithms to solve problems in trigonometry when ratios are equal to each other and one is looking to find an unknown value.

Show that if $x : y = z : w$ then Nlog x + Nlog w= Nlog y + Nlog z

Is Nlog AB equal to Nlog A + Nlog B?

What is Nlog(1)?

Henry Briggs (1561-1631) convinced Napier to change his logarithm so that log(1)=0 and log(10)=1 but the change was not made before Napier died. Briggs' table of logarithms completed in 1628 were widely adopted quickly so that Laplace said that logarithms "by shortening labors, doubled the life of the astronomer."