Numerical Analysis

Math 370 Spring 2009 MWF 11:30am - 12:25pm Fowler 110
 http://faculty.oxy.edu/ron/math/370/09/

Worksheet 19

SUMMARY Piece-Wise Polynomial Interpolation; Introduction to Splines
READING Recktenwald, Sec 9.1, pp 455–468; Mathews & Fink Section 5.3

Piecewise Polynomial Interpolation

Instead of using one interpolating polynomial of high degree, there is also the idea of using a lot of polynomials of small degree to do interpolation. For example, with the example of producing cheap and dirty sines, we said that we would use a number of linear interpolations between consecutive nodes. We say that these are **piecewise polynomials** because the "pieces" (the individual linear interpolating polynomials) are tied together at the "knots" (also known as the nodes). These pieces are also known as **splines**.

Piecewise Linear Interpolation

1. Sketch a figure of piecewise linear interpolation for the following data

2. Given accurate data (x_k, y_k) for $k = 0$ to n come up with the equation for a typical piecewise linear interpolant $s_k(x)$

3. What is the relationship between $s_k(x)$ on the kth interval and s_{k+1} on the $(k+1)$th interval? [HINT: think about their common point.]

4. What are some drawbacks of doing piecewise linear interpolation instead of doing piecewise polynomial interpolation with a higher degree polynomial?

The most common piecewise polynomial interplation is done with polynomials of degree **three**, thus leading to the famous **cubic spline** problem.

The Cubic Spline Problem

Given that $s_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + d_k(x - x_k)^3$ and that we have $N + 1$ data points (x_k, y_k) how many cubic splines will we use to interpolate the entire interval from x_0 to x_N? How many unknown constants do we have to compute (i.e find) in order to define all of these $s_k(x)$?

5. How many equations do we have to solve for to find these unknown constants?
Write them down.

6. What is a **natural spline**?

7. What is a **clamped spline**?

There isn't a known error formula for the **natural cubic spline**, but there is one for the clamped cubic spline:
Let M be a bound for $f^{(4)}(x)$ on $[a, b]$. Then if S is the unique clamped spline interpolant with respect to the nodes $a = x_0 < x_1 < \ldots < x_n = b$ then

$$\max_{a \leq x \leq b} |f(x) - S(x)| \leq \frac{5M}{384} \max_{0 \leq j \leq n-1} (x_{j+1} - x_j)^4$$

Matlab implementation of cubic splines

8. Let's compute the clamped cubic spline for the data (1,1), (2,3), (3,2), (4,4). We want the derivatives at the end points to be 0. Look at the Matlab `splint.m` and/or `splintFE.m` programs.

9. What commands would you have to use for the natural spline?

10. Write down the commands necessary to plot the cubic spline AND the data. Choose different colors for the data and the piecewise interpolants.