$\mathbf{N}$umerical $\mathbf{A}$nalysis

Worksheet 15

SUMMARY Implementing Polynomial Interpolation
READING Recktenwald, pp. 521–538; Mathews & Fink Section 4.3-4.4.

General Interpolation Problem

The general interpolation problem involves $N + 1$ data points (x_0, y_0), (x_1, y_1) ... (x_N, y_N) which we assume we know with very high accuracy. The general problem is to find an interpolating function $\mathcal{F}(x)$ can be expressed as a linear combination of **basis functions** $\phi_i(x)$

$$\mathcal{F}(x) = a_0\phi_0(x) + a_1\phi_1(x) + \ldots + a_n\phi_n(x)$$

where $a_0, a_1, a_2, \ldots, a_n$ are coefficients which need to be determined.

We will only be considering **polynomial** basis functions, because of the Weierstrass Approximation Theorem and also because they are easy to evaluate and manipulate.

Previously we defined the Lagrange Polynomials and have been using them as our basis functions.

$\boxed{\text{Exercise}}$

Write down **all** the Lagrange Polynomials of degree 1 and degree 2 for the nodes x_0, x_1 and x_2.

Uniqueness
NOTE: For $N+1$ data points the interpolating polynomial of degree N is unique.
However, there are other forms this interpolating polynomial can be written if we use different basis functions.

Using MATLAB to Do Lagrange Interpolation

In the NMM Toolbox we have the m-file `lagrint.m` which contains an implementation of using Lagrange Polynomials as basis functions to do interpolation.

Exercise

Consider the data below for the Bessel's Function $J_0(x)$. Bessel's Functions are special functions which arise in the solution of partial differential equations in cylindrical coordinates.

x	1.0	1.3	1.6	1.9	2.2
$J_0(x)$	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

What is the degree of the unique interpolating polynomial for this data?

Since this would not be fun to compute by hand, let's try and use MATLAB. Use `lagrint.m` to do Lagrange Interpolation to estimate $J_0(1.5)$

Lagrange basis polynomials of degree 1 through 4

Interpolation with other basis functions

Let us try using the monomials (i.e. functions which look like x^n, for $n \geq 0$) as basis functions. In other words we will be trying to estimate our unknown function $f(x)$ (for which we only have data on $n + 1$ points) using the interpolant $\mathcal{F}(x)$ given by

$$\mathcal{F}(x) = a_0 \cdot 1 + a_1 \cdot x + a_2 \cdot x^2 + \dots a_n \cdot x^n$$

For these basis functions finding the set of coefficients $a_i, 0, 1, 2, \dots n$ involves solving _____

| EXAMPLE |

1. Let's try and find the unique interpolating polynomial for the data from the mystery function of $(-1, 1/2)$, $(0, 1)$ and $(1, 2)$. What are the equations for the coefficients a_0, a_1 and a_2?

2. Write down your system in matrix equation form.

3. Solve the system.

Exercise

4. In general, if your points had been (x_0, y_0), (x_1, y_1) and (x_2, y_2) write down the system you would have to solve in matrix equation form.

This is a special kind of matrix called a **Vandermonde matrix.** Because of that there is a special MATLAB command called `vander` which, given a vector c will generate the associated Vandermode matrix.

GroupWork

5. Try and use MATLAB to extrapolate the value of the mystery function at $x = 1.5$. (You may want to use the `polyval` function.)