Numerical Analysis

Math 370 Spring 2009 MWF 11:30am - 12:25pm Fowler 110

 `http://faculty.oxy.edu/ron/math/370/09/`

Worksheet 9

SUMMARY Other Root-finding Methods (False Position, Newton's and Secant)
READING Recktenwald, 6.1.1 (240-250); Mathews & Fink 2.2-2.3 (62-90)

RECALL
Write down what steps you had to take in order to use `bisect.m` to solve the equation $2552 - 30d^2 + d^3 = 0$

$d =$

Assessing Bisection
What are some good features of the bisection algorithm?

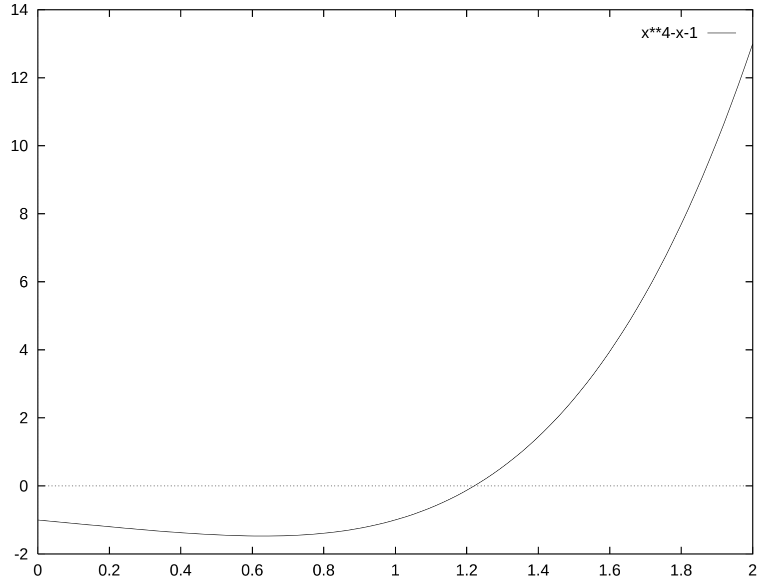
What are some drawbacks to the bisection algorithm?

False Position Method (Regula Falsi)

The method of False Position is another ancient method of computing the solution of equations of one variable. It is very similar to the bisection method in that it too is a bracketing method, except that False Position uses the **value** of the function at the end points to help determine where the next bracket occurs. Draw a line connecting $(a, f(a)$ and $(b, f(b))$ and the new bracket will be formed using the x-intercept of this line.

Sketch a picture of the iterative process of the **False Position** algorithm, below:

False Position visually



Exercise

Let's try and derive the iterative step used in the False Position algorithm.
If we have a bracket $[a_n, b_n]$ how do we find the value p_n which is the next estimate of the root?

False Position Algorithm	Bisection Algorithm
	INPUT $f(x)$,a,b, ITMAX, TOL
	LET IT=0
	WHILE $IT < ITMAX$
	LET $p = \frac{1}{2}(a + b)$
	IF $f(p) < TOL$ ACCEPT P AS ROOT, RETURN
	IF $f(p) > TOL$ (CHECK BRACKET
	IF $F(p) * F(B) < 0$
	LET $a = p$ (Choose [p,b] bracket)
	IF $F(p) * F(B) > 0$
	LET $b = p$ (Choose [a,p] bracket)
	IT = IT + 1
	END WHILE
	OUTPUT $p, f(p)$

EXAMPLE

Use False Position to solve the same equation $f(d) = 2552 - 30d^2 + d^3 = 0$ you previously solved using Bisection and see if there is a difference in the number of steps False Position takes to converge versus Bisection. In `s:\Math Courses\Math370\Spring2009\rootfind` there is an implementation of the False Position algorithm in MATLAB . Can you see the similarities to the Bisection Algorithm?

Assessing False Position

What are some good features of the false position algorithm?

What are some drawbacks to the false position algorithm?

The Newton-Raphson Algorithm

Bisection and False Position are both **globally convergent** algorithms, because, given a bracket which contains a solution, they both will find the solution, eventually.

Newton's Method (and the Secant Method) are very different from these methods, in that instead of needing a bracket where the solution exists [i.e. continuous function has values at the bracket endpoints have opposite sign] one needs a **single** guess of the solution, which has to be "close" to the exact answer, in order for these **locally convergent** to get the solution.

A Derivation of Newton's Method

Write down the first 3 terms of a Taylor expansion of $f(x)$ about the point $(p_0, f(p_0))$

Evaluate this function at the root, the point $(p_1, 0)$ and solve for p_1

This is the iterative step for Newton's Method

$p_{n+1} =$

Pseudocode for Newton's Method

INPUT: x_0, $f(x)$, $f'(x)$
FOR k = 1 to NSTEPS
$x_{k+1} = x_k - f(x_k)/f'(x_k)$
OUTPUT k, x_k, $f(x_k)$
IF ''CONVERGED'', STOP
END

EXAMPLE

Consider the function $f(x) = x^2 - A$, where $A > 0$

Compute the Newton iterative step using the above function $f(x)$

Simplify it, so that it look like $x_{n+1} = \dfrac{x_n + A/x_n}{2}$. Recognize this iteration?

Exercise

Let $A = 2$ and $x_0 = 1$. Find x_3

Secant Method

The secant method is very similar to Newton's method, except that instead of actually computing the derivative, one approximates it using a difference quotient. This ends up in making the iterative step look algebraically identical to the one for the False Position method.

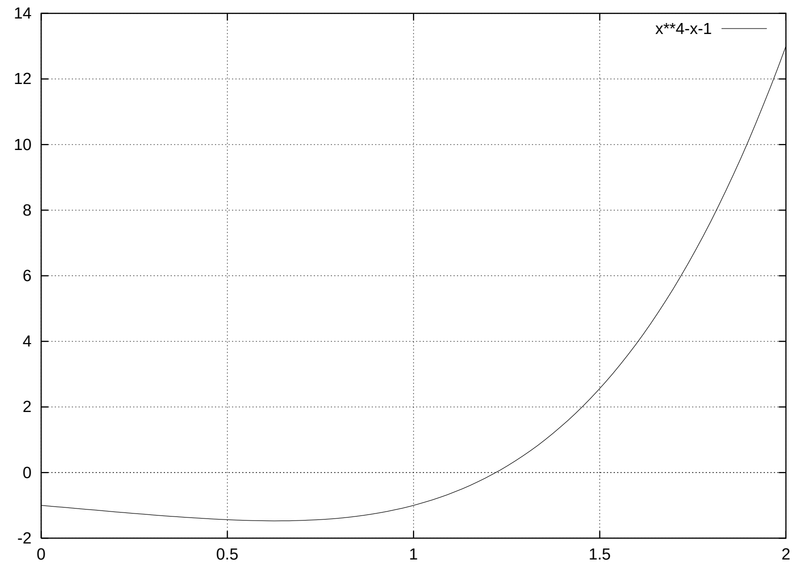
Exercise

We will write down the **Secant Method iterative step** below

$p_{n+1} =$

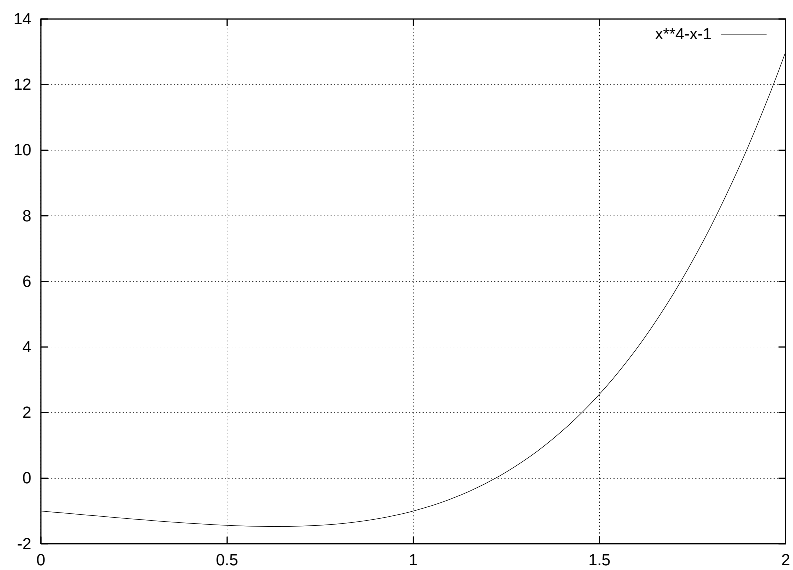
If the iterative step is identical to False Position, how come the Secant Method is not just called the False Position method? Look at the picture...

Secant Method, visually



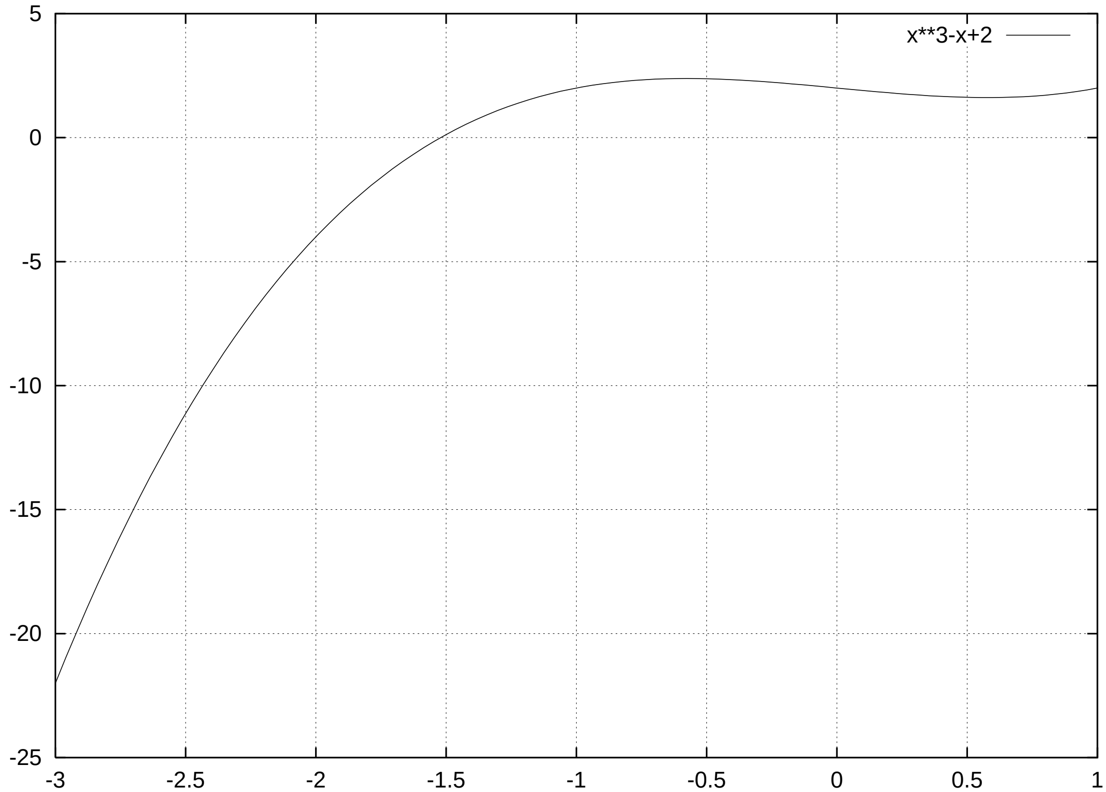
False Position Method, visually

Now, let's recall what False Position look like, visually...

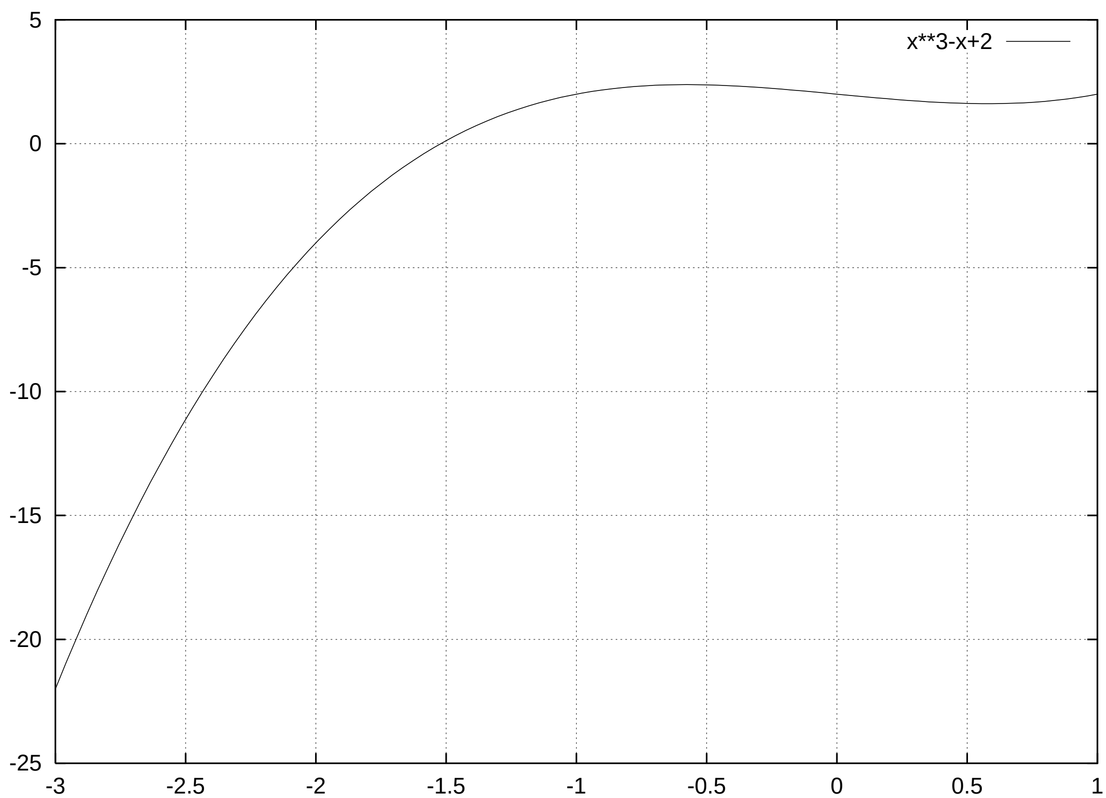


GroupWork

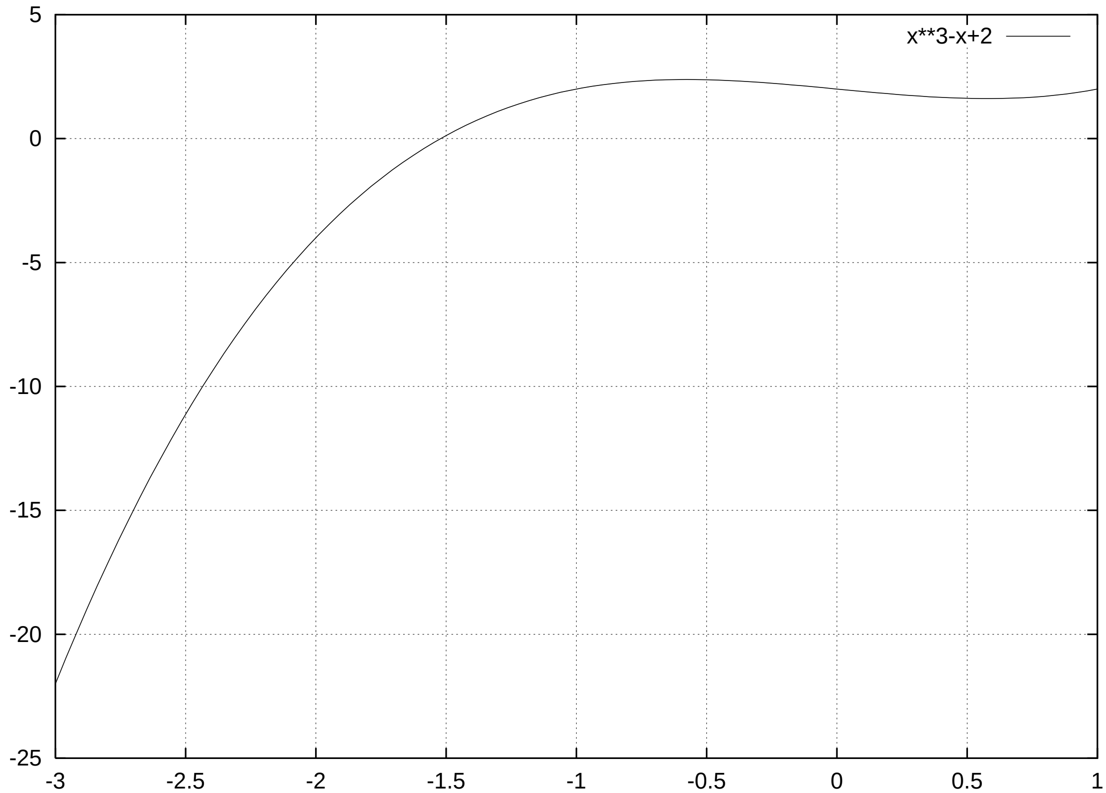
Consider $f(x) = x^3 - x + 2$. On the following figure, draw on the graph the set of approximations to the zero, i.e. $\{p_k\}$, due to Newton's Method, if you start at $p_0 = 1$



On the following figure, draw on the graph the set of approximations to the zero, i.e. $\{p_k\}$, due to the Bisection Method, if you start with the bracket $[-3, 1]$



On the following figure, draw on the graph the set of approximations to the zero, i.e. $\{p_k\}$, due to the False Position Method, if you start with the bracket $[-3, 1]$



On the following figure, draw on the graph the set of approximations to the zero, i.e. $\{p_k\}$, due to the Secant Method, if you start with the bracket $p_0 = -3$ and $p_1 = 1$