Math 370 Fall 2002 **SHOW ALL YOUR WORK** Quiz Five

1. *[4 pts]* Use **Newton's Method**, $p_{n+1} = p_n - \dfrac{f(p_n)}{f'(p_n)}$, to obtain the value of $\sqrt[3]{4}$ to within 3 decimal places by finding the root of $f(x) = x^3 - 4$ using an initial guess of $p_0 = 4$. Show the details of your calculation of p_1, p_2 and p_3 and then just the values of the subsequent p_n values. $p_0 = 4$ $f(x) = x^3 - 4$ $f'(x) = 3x^2$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 4 - \frac{4^3-4}{3\cdot 4^2} = 4 - \frac{60}{48} = 2.75$$

$$p_2 = 2.75 - \frac{2.75^3 - 4}{3\cdot 2.75^2} = 2.0096$$

$$p_3 = 2.0096 - \frac{2.0096^3 - 4}{3\cdot 2.0096^2} = 1.6699$$

$p_4 = 1.5914$

$p_5 = 1.5874$

~~$p_6 = 1.5874$~~

$p_5 - p_4 = 4\times10^{-4} < 5\times10^{-4}$

2. Consider a new method of finding a root of an equation, called **The Lazy Newton** or "Fixed Slope" method. This is similar to Newton's Method except that instead of taking the derivative at EVERY step, one computes the derivative once at the point of the initial guess p_0 and uses only that derivative in every subsequent iteration.

   The general formula for the **Lazy Newton** method is: $p_{n+1} = p_n - \dfrac{f(p_n)}{f'(p_0)}$

(a) *[4 pts]* Indicate on the graph of the function $f(x) = x^3 - 4$ below what the first few approximations to the root, p_1, p_2, p_3 will look like, using the Lazy Newton method, given that $p_0 = 4$. Make sure you indicate how you computed p_1, p_2 and p_3.

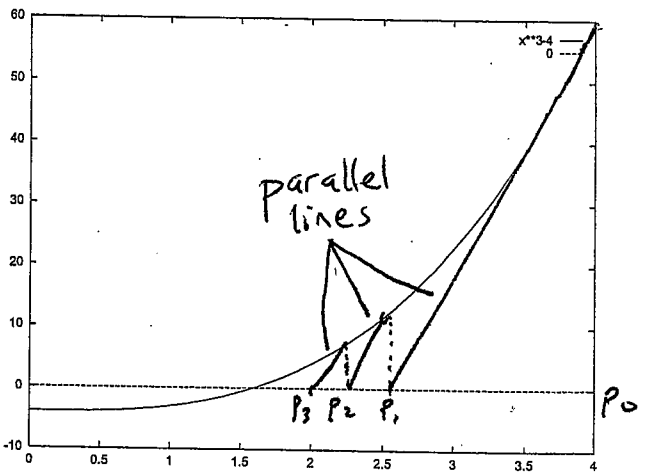
$p_0 = 4$ $f(4) = 60$ $f'(4) = 48$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 4 - \frac{f(4)}{f'(4)}$$

$p_1 = 2.75$

$p_2 = 2.75 - \dfrac{(2.75^3 - 4)}{48} = 2.4001$

$p_3 = 2.4001 - \dfrac{2.4001^3 - 4}{48} = 2.1954$



(b) *[2 pts]* Use Lazy Newton's Method to find the value of $\sqrt[3]{4}$ to within 3 decimal places. Which method do you expect to be faster, Newton's or Lazy Newton's? Which one "converges" faster to $\sqrt[3]{4}$? Newton's takes 4 steps.

Lazy Newton's take 31 steps to approach $|p_n - p_{n-1}| < 5\times10^{-4}$

$p_{31} = 1.590$

NOTE: $1.590^3 - 4 = 0.018$

$1.587^3 - 4 = -0.003$

Newton's is faster, unsurprisingly