

QUIZ 3

Numerical Analysis

Name: _____

Friday February 13

Prof. Ron Buckmire

Time Begun: _____

Time Ended: _____

Topic : Appreciating Quadratic Convergence

The idea behind this quiz is for you to give you an opportunity to demonstrate your understanding of the relative rates of convergence of functions, grow more familiar with \mathcal{O} and o notation (sometimes called Bachmann-Landau symbols), and to practice your ability to find limits.

Reality Check:

EXPECTED SCORE : _____/10

ACTUAL SCORE : _____/10

Instructions:

0. Please look for a hint on this quiz posted to faculty.oxy.edu/ron/math/370/09/
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Wednesday February 18**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. (a.) (1 point.) For each pair of integers k and m determine an integer N for which

$$\left| \frac{1}{N^k} - 0 \right| < 10^{-m}.$$

(b.) (3 points). Show that for any positive integer k , the sequence defined by $p_n = \frac{1}{n^k}$ converges **linearly** to $p_\infty = 0$. Write a mathematical sentence (i.e. an equation) which relates the sequence p_n , the sequence's limit and one of the Bachmann-Landau symbols \mathcal{O} or o with a gauge sequence of your choice.

2. (4 points) Show that $q_n = \frac{1}{10^\alpha n}$ has an asymptotic rate of convergence of $\alpha!$ (In other words, if $\alpha = 3$, the sequence will be cubically convergent, etc.)

(b.) (2 points) Which element of the sequence $r_n = \frac{1}{10^{3^n}}$ will be within 10^{-12} of its limit?