**1.** Consider each of the following expressions $f(h)$ as $h \to 0$. Express each of them in the form $f(h) = L + \mathcal{O}(h^\alpha)$ with the "best" (most accurate) integer values of $\alpha > 0$. For each problem write down a value of $\alpha$ and $L$.

**a.** $e^{h^2}$

$\lim_{h\to 0} e^{h^2} = e^0 = \boxed{1 = L}$

$e^{\square} = 1 + \square + \frac{\square^2}{2!} + \frac{\square^3}{3!} + \dots$

$e^{h^2} = 1 + h^2 + \frac{(h^2)^2}{2!} + \dots = 1 + \mathcal{O}(h^2)$ $\boxed{\alpha = 2}$

$\lim_{h\to 0} \frac{e^{h^2}-1}{h^2} \overset{L'h}{=} \lim_{h\to 0} \frac{2he^{h^2}}{2h} \overset{L'h}{=} \lim_{h\to 0} e^{h^2} = 1$ ✓

**b.** $(1-h^2)^{-1}$

$\lim_{h\to 0} \frac{1}{1-h^2} = \boxed{1 = L}$ $\boxed{\alpha = 2}$

$\frac{1}{1-\square} = 1 + \square + \square^2 + \square^3 + \dots$

$\frac{1}{1-h^2} = 1 + h^2 + (h^2)^2 + (h^2)^3 + \dots = 1 + \mathcal{O}(h^2)$

$= 1 + h^2 + h^4 + h^6 + \dots$

$\lim_{h\to 0} \frac{\frac{1}{1-h^2} - 1}{h^2} = \lim_{h\to 0} \frac{\frac{h^2}{1-h^2}}{h^2} = \lim_{h\to 0} \frac{1}{1-h^2} = 1$ ✓

**c.** $\frac{\ln(1+h)}{h}$

$\lim_{h\to 0} \frac{\ln(1+h)}{h} = \lim_{h\to 0} \frac{\frac{1}{1+h}}{1} = \lim_{h\to 0} \frac{1}{1+h} = \boxed{1 = L}$ $\boxed{\alpha = 1}$

$\ln(1+\square) = \square - \frac{\square^2}{2} + \frac{\square^3}{3} - \dots$

$\ln(1+h) = h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + \dots$

$\frac{\ln(1+h)}{h} = 1 - \frac{h}{2} + \frac{h^2}{3} + \dots = 1 + \mathcal{O}(h)$

$\lim_{h\to 0} \frac{\frac{\ln(1+h)}{h} - 1}{h} = \lim_{h\to 0} \frac{\ln(1+h) - h}{h^2} = \lim_{h\to 0} \frac{\frac{1}{1+h} - 1}{2h} = \lim_{h\to 0} \frac{\frac{-1}{(1+h)^2}}{2} = -\frac{1}{2}$

**d.** $\cos(h^2)$

$\lim_{h\to 0} \cos(h^2) = \boxed{1 = L}$ $\boxed{\alpha = 4}$

$\cos(\square) = 1 - \frac{\square^2}{2!} + \frac{\square^4}{4!} - \dots$

$\cos(h^2) = 1 - \frac{(h^2)^2}{2!} + \frac{(h^2)^4}{4!} - \dots$

$= 1 - \frac{h^4}{2} + \frac{h^8}{24} + \dots = 1 + \mathcal{O}(h^4)$

$\lim_{h\to 0} \frac{\cos(h^2) - 1}{h^4} = \lim_{h\to 0} \frac{-2h\sin(h^2)}{4h^3} = \lim_{h\to 0} \frac{\sin(h^2)}{-2(h^2)} = \lim_{h\to 0} \frac{2h\cos(h^2)}{-4h} = \lim_{h\to 0} \frac{\cos(h^2)}{-2} = -\frac{1}{2}$

**e.** $1 + \sin(h^3)$

$\lim_{h\to 0} 1 + \sin(h^3) = \boxed{1 = L}$

$\sin\square = \square - \frac{\square^3}{3!} + \frac{\square^5}{5!} - \dots$

$\sin(h^3) = (h^3) - \frac{(h^3)^3}{3!} + \frac{(h^3)^5}{5!} - \dots = h^3 - \frac{h^9}{6} + \dots$

$1 + \sin(h^3) = 1 + h^3 - \frac{h^9}{6} + \dots = 1 + \mathcal{O}(h^3)$ $\boxed{\alpha = 3}$

$\lim_{h\to 0} \frac{1 + \sin(h^3) - 1}{h^3} = \lim_{h\to 0} \frac{\sin(h^3)}{h^3} = \lim_{h\to 0} \frac{3h^2\cos(h^3)}{3h^2} = \lim_{h\to 0} \cos(h^3) = 1$