# Numerical Analysis

Math 370 Spring 2009 MWF 11:30am - 12:25pm Fowler 110
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## Homework Set 4

5 questions+journal, 50 points

**ASSIGNED: Wed Apr 8 2009** **DUE: Wed Apr 22 2009**

1. (*9 points*) **Recktenwald, Chapter 9, #7, page 512.**
Least Squares Fitting Problems with only one undetermined coefficient lead to particularly simple computational procedures. We derived the solution of the normal equations when $P(x) = ax + b$. In this problem derive the equations for finding the (scalar) coefficient $c$ of the following equations, assuming that $\vec{x}$ and $\vec{y}$ are known vectors of data of length $m$ for the following relationships:

   **(a.)** $y = P(x) = cx$

   **(b.)** $y = P(x) = cx^2$

   **(c.)** $y = P(x) = x^c$

2. (*10 points*) **Mathews & Fink, Section 5.2, #7, page 276-277.** The function $P(t) = \dfrac{L}{1 + Ce^{At}}$ can represent a population $P$ bounded by a limiting value $L$.
**(a)** Find $A$ and $C$ for the following data sets, assuming that $L = 8 \times 10^8$ and estimate the population in the Year 2000. Compare your estimate to the official number available from the U.S. Census.
**(b)** Plot the data and the curves of best fit on the same axes using MATLAB .

**50-Year Data**

| Year | $t_k$ | $P_k$ |
|---|---|---|
| 1800 | -10 | 5.3 |
| 1850 | -5 | 23.2 |
| 1900 | 0 | 76.1 |
| 1950 | 5 | 152.3 |

**20-Year Data**

| Year | $t_k$ | $P_k$ |
|---|---|---|
| 1900 | 0 | 76.1 |
| 1920 | 2 | 106.5 |
| 1940 | 4 | 132.6 |
| 1960 | 6 | 180.7 |
| 1980 | 8 | 226.5 |

3. (*8 points*) Consider the data (8,16.63553), (8.1,17.61549), (8.3,17.56492), (8.6,18.50515), (8.7,18.82091). Use Lagrange Interpolation of degree 1, 2, 3 and 4 to interpolate the data at the point $x = 8.5$. Which interpolant do you expect to be most accurate? Is it? (HINT: use the `lagrint` m-file)

4. (*6 points*) Consider a function $f$ defined on [a,b] with nodes $a = x_0 < x_1 < x_2 = b$.
A quadratic spline interpolating function $S(x)$ consists of the polynomials $S_0(x) = a_0 + b_0(x - x_0) + c_0(x - x_0)^2$ on $[x_0, x_1]$ and $S_1(x) = a_1 + b_1(x - x_1) + c_1(x - x_1)^2$ on $[x_1, x_2]$. This Quadratic Spline obeys the conditions
   **(i.)** $S(x_0) = f(x_0), \quad S(x_1) = f(x_1) \quad S(x_2) = f(x_2)$
   **(ii.)** $S_0'(x_1) = S_1'(x_1)$

   Show that these conditions lead to 5 equations in the 6 unknowns $a_0, b_0, c_0, a_1, b_1, c_1$. Come up with another condition that you would like your quadratic spline to obey and see if this makes the solution of your system unique.

5. (*7 points*) Consider the points (-1,1/2), (0,1) and (1,2). Consider the functions $P_1(x) = ax + b$ and $P_2(x) = \dfrac{1}{cx+d}$. Find $P_1$ and $P_2$ which minimize the least square error between the functions and the data.

   **(a.)** Which function does a better job of fitting the data? Plot the data and the curves $P_1$ and $P_2$

   **(b.)** Use an interpolant of appropriate degree and plot it through the data.

   **(c.)** The exact function which generates this data is $y = 2^x$. Which function gets closest to approximating the actual function at $x = 2$?

## JOURNAL ENTRY

(*10 points*) Comment on your understanding of the differences between the "best fit" problem and the "interpolation" problem. What are the similarities and differences? Do you feel confident that given some data you could use MATLAB to produce a solution to both of these problems?

**Self-Assessment:** Discuss your thoughts about the class as a whole. What were the topics that you understood the most and which topics did you understand the least? What was the most effective part of the class? What was the least effective? What changes would you recommend to the class for when it is taught again?

## NOTES

You are **strongly** encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work. You should indicate on your neatly-written solution manuscripts which students you collaborated with. If you encounter difficulty, you should ask questions on the online message board at `http://moodle.oxy.edu` , or via the *Numerical Analysis* class email list at `math370-L@oxy.edu`, or come see me in my office.