Numerical Analysis

Math 370 Spring 2009 MWF 11:30am - 12:25pm Fowler 110

 http://faculty.oxy.edu/ron/math/370/09/

Homework Set 3

5 questions+journal= 40+10 points

ASSIGNED: Mon Mar 23 2009 **DUE: Mon Apr 06 2009**

1. (*8 points*) Show that the following identities are true for all n dimensional vectors $\vec{x}$

(a.) Show that $|\vec{x}|_\infty \leq |\vec{x}|_2 \leq |\vec{x}|_1$

(b.) If $\vec{a} = (1, -2, 4, -8, 16)^T$ and $\vec{b} = (1, 1, 1)^T$ show that the vectors $\vec{a}$ and $\vec{b}$ satisfy the identity from part (a).

2. (*8 points*) **Mathews & Fink, page 182, #9**. Show that Jacobi iteration for a 3×3 linear system is simply a special case of fixed point iteration $\vec{x}^{(k+1)} = \vec{G}(\vec{x}^{(k)})$ given on page 173 (equation #15). Furthermore, verify that if the coefficient matrix from a 3×3 linear system is strictly diagonally dominant, then the condition on page 173 (equation #17) is satisfied.

3. (*8 points*) **Mathews & Fink, page 182, #8**. Consider the non-linear system

$$\begin{aligned} 0 = f_1(x, y) &= x^2 + y^2 - 2 \\ 0 = f_2(x, y) &= xy - 1 \end{aligned}$$

(a.) Verify that the solutions are $(1, 1)$ and $(-1, -1)$

(b.) Sketch a graph of the functions to indicate the points of intersection

(c.) What difficulties arise if you try to use Newton's Method for Systems to find the solutions?

(d.) Use a different numerical method to find both solutions to the nonlinear system.

4. (*8 points*) Consider the system

$$\begin{aligned} 5x - y &= 4 \\ -x + 5y + -2z &= 2 \\ -2y + 5z &= 3 \end{aligned}$$

(a.) Use Gauss-Seidel and Jacobi Iteration to solve the system using a tolerance of 10^{-8}

(b.) Mathematically explore which value of ω solves the system the fastest using SOR.

(c.) Show that the given system has a Jacobi matrix (i.e. $D^{-1}(L+U)$) which has a spectral radius of $\frac{1}{\sqrt{5}}$.

(d.) Use your result in (c) to confirm that the theoretical optimal relaxation parameter for SOR is 1.05573.

5. (*8 points*) Linear systems are just a special case of nonlinear systems. Let's see what happens if we use nonlinear solvers on linear systems.

(a.) Rewrite the previous system so that it is a fixed point problem. (HINT: just change the system to be $A\vec{x} - \vec{b} + \vec{x} = \vec{G}(\vec{x})$.)

(b.) Use Successive Substitution and Seidel iteration to find the solution of the system (i.e. the fixed point of $\vec{G}$) to within a tolerance of 10^{-8}

(c.) Use Newton's Method for System to also solve the same system to the same tolerance. Make sure you use the correct $\vec{F}(\vec{x})$ and Jacobian to ensure you are solving the same problem you solved in part (b).

Liberal use of the `diary` command together with printouts of any Matlab m-files you write are expected to be included.

JOURNAL ENTRY

(*10 points*) Now that you have used Jacobi, Gauss-Seidel, Seidel, Successive Substitution, Newton's Method and SOR to solve the same problem discuss which method you prefer, and which method seems the most useful when faced with solving a system of multiple equations in multiple variables. Would it matter if it were a nonlinear or linear system? Do you see any relationships or similarities between the methods?
Self-Assessment: In addition, write a paragraph describing your expectations for the term project. The Term Project will most likely involve applying the numerical methods you now know to a new mathematical domain you have not seen before. How will you work with your partner to ensure that a fair division of work occurs? How would you like to have your Term Project partner chosen? By you or by me?

BONUS

Inspired by **Mathews & Fink, page 185 #5 and #6**. Consider the non-linear system

$$
\begin{aligned}
0 &= 7x^3 - 10x - y - 1 \\
0 &= 8y^3 - 11y + x - 1
\end{aligned}
$$

Use MATLAB to sketch the graphs of both curves on the same coordinate system. Use the graph to verify that there are nine points where the graphs intersect. Using the graph, *estimate* the points of intersection. Use these estimates and MATLAB programs to approximate the coordinates of the points of intersection to 9 decimal places.
Now consider the fixed-point form of the above system

$$
\begin{aligned}
x &= \frac{7x^3 - y - 1}{10} \\
y &= \frac{8y^3 + x - 1}{11}
\end{aligned}
$$

No matter what starting value you use, this iterative scheme only converges to one of the 9 intersection points you found early. Explain this behavior.

NOTES

You are **strongly** encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work. You should indicate on your neatly-written solution manuscripts which students you collaborated with. If you encounter difficulty, you should ask questions on the online message board at `http://moodle.oxy.edu` , or via the *Numerical Analysis* class email list at `math370-L@oxy.edu`, or come see me in my office.