HW SET 3

1(a) $\|\vec{x}\|_\infty = \max_i |x_i|$

$\|\vec{x}\|_1 = \sum_{i=1}^{n} |x_i|$

$\|\vec{x}\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$

We know $\sqrt{a} + \sqrt{b} \geq \sqrt{a+b}$ for all $a, b \geq 0$

since $a + b + \sqrt{a}\cdot\sqrt{b}\,2 \geq a+b$

$\sqrt{\sum_{i=1}^{n} x_i^2} \leq \sum_{i=1}^{n} \sqrt{x_i^2} = \sum_{i=1}^{n} x_i \leq \sum_{i=1}^{n} |x_i|$

$\|\vec{x}\|_2 \leq \|\vec{x}\|_1$

$\|\vec{x}\|_\infty \leq \sqrt{\left(\max_{1 \leq i \leq n} |x_i|\right)^2}$

$\|\vec{x}\|_\infty \leq \sqrt{\max_i x_i^2 + \sum_{j=1, j \neq i}^{n} x_j^2} = \|\vec{x}\|_2$ where x_i is max element

So, $\|\vec{x}\|_\infty \leq \|\vec{x}\|_2$

and $\|\vec{x}\|_2 \leq \|\vec{x}\|_1$

(b) $\|\vec{x}\|_1 = |x_1| + |x_2| + |x_3| + \ldots + |x_n| \leq \overbrace{x_{max} + x_{max} + \ldots + x_{max}}^{n} = n\, x_{max}$

let $x_{max} = \max_{1 \leq i \leq 1} |x_i|$

$\|\vec{x}\|_1 \leq n \|\vec{x}\|_\infty$

(c) $\|\vec{x}\|_2^2 = x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2 \leq \overbrace{x_{max}^2 + x_{max}^2 + \ldots + x_{max}^2}^{n} = n\, x_{max}^2$

$\|\vec{x}\|_2 \leq \sqrt{n}\, x_{max} = \sqrt{n}\, \|\vec{x}\|_\infty$

1 (a) $\|\vec{a}\|_1 = 1 + |-2| + |4| + |-8| + |16|$

$= 1 + 2 + 4 + 8 + 16$

$= 31$

$\|\vec{a}\|_2 = \sqrt{1^2 + 2^2 + 4^2 + 8^2 + 16^2}$

$= \sqrt{1 + 4 + 16 + 64 + 256}$

$= \sqrt{341}$

$\|\vec{a}\|_\infty = \max(1, 2, 4, 8, 16)$

$= 16$

$\|\vec{b}\|_1 = |1| + |1| + |1| = 3$

$\|\vec{b}\|_2 = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\|\vec{b}\|_\infty = \max 1 = 1$

$\|\vec{a}\|_\infty \leq \|\vec{a}\|_2 \leq \|\vec{a}\|_1$

$16 \leq \sqrt{341} \leq 31$

$\|\vec{b}\|_\infty \leq \|\vec{b}\|_2 \leq \|\vec{b}\|_1$

$1 \leq \sqrt{3} \leq 3$

$\|\vec{a}\|_1 \leq n \|\vec{a}\|_\infty$

$31 \leq 4 \cdot 16 = 64$

$\|\vec{b}\|_1 \leq n \|\vec{b}\|_\infty$

$3 \leq 3 \cdot 1 = 3$

$\|\vec{a}\|_2 \leq \sqrt{n} \|\vec{a}\|_\infty$

$\sqrt{341} \leq \sqrt{4} \cdot 16 = 32$

$\|\vec{b}\|_2 \leq \sqrt{n} \|\vec{b}\|_\infty$

$\sqrt{3} \leq \sqrt{3} \cdot 1$

2.

$$f_1(x,y) = x^2+y^2-2 = 0$$
$$f_2(x,y) = xy-1 = 0$$

We could write this as a one-variable system

$xy-1=0 \Rightarrow y = \frac{1}{x}$

$$x^2+\left(\frac{1}{x}\right)^2 - 2 = 0$$

$$x^2 + \frac{1}{x^2} - 2 = 0$$

$$x^4+1-2x^2 = 0$$

$$(x^2-1)^2 = 0$$

$$x^2 = 1$$

$$x = \pm 1 \Rightarrow y = \pm 1$$

$(1,1)$ and $(-1,-1)$

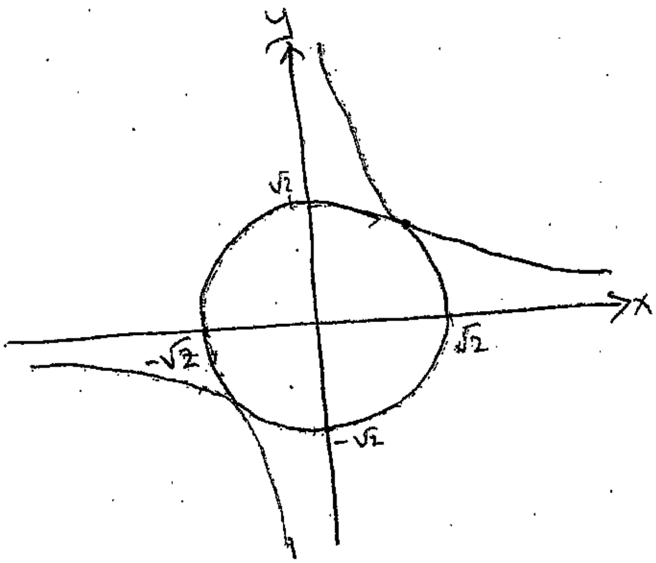
Could use Newton's Method but it will converge quite slowly.

3.
(a) $\underline{f}(\vec{x}) = \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix} = \begin{pmatrix} x^2 + y^2 - 2 \\ xy - 1 \end{pmatrix}$

$\underline{f}(1,1) = \begin{pmatrix} f_1(1,1) \\ f_2(1,1) \end{pmatrix} = \begin{pmatrix} 1^2 + 1^2 - 2 \\ 1 \cdot 1 - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\underline{f}(-1,-1) = \begin{pmatrix} f_1(-1,-1) \\ f_2(-1,-1) \end{pmatrix} = \begin{pmatrix} -1^2 + -1^2 - 2 \\ (-1)(-1) - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

(b)



(c) $J(x,y) = \begin{pmatrix} 2x & 2y \\ y & x \end{pmatrix}$

$J(1,1) = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$ $\quad J(-1,-1) = \begin{pmatrix} -2 & -2 \\ -1 & -1 \end{pmatrix}$

The Jacobian is singular at (1,1) and (-1,-1)
thus you can't use Newton's Method for systems
to solve the system.

$\Delta x = -\underline{\underline{J}}^{-1}(x) \cdot \vec{f}(\vec{x})$

J^{-1} doesn't exist

4. $5x - y = 4$

$-x + 5y - 2z = 2$

$-2y + 5z = 3$

$$\begin{pmatrix} 5 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \Rightarrow A\vec{x} = \vec{b}$$

$A = D - L - U$

$$D = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \quad L+U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

$$T_j = D^{-1}(L+U) = \begin{pmatrix} 0 & 1/5 & 0 \\ 1/5 & 0 & 2/5 \\ 0 & 2/5 & 0 \end{pmatrix}$$

jacobi(a, b, 10^(-8))

gseidel(a, b, 10^(-8))

Try $\omega = 1.0$

$\omega = 1.5$

$\omega = 1.25$

Best result occurs with

$\omega = 1.07$

```
type gee4


function g=gee4(x)
g(1) = 5*x(1)-x(2) - 4 + x(1);
g(2) = -x(1) + 5*x(2) - 2*x(3) - 2 + x(2);
g(3) = -2*x(2) + 5*x(3) - 3 + x(3);
type gee


function [J,f] = gee(x)
%
%
%
f(1) = 5*x(1)-x(2)-4;
f(2) = -x(1)+5*x(2)-2*x(3)-2;
f(3) = -2*x(2)+5*x(3)-3;
J = [5 -1 0; -1 5 -2 ; 0 -2 5];
f = [f(1) ; f(2) ; f(3) ];


succsub('gee4',[0 0 0]')

  k       x(1)       x(2)       norm(dx)
  0     0.00000    0.00000
  0     0.00000   ??? Error using ==> -
Matrix dimensions must agree.

Error in ==> q:\mfiles\math370\nmm\rootfind\succsub.m
On line 32  ==>    dx = norm(x-f,2);

succsub('gee4',[0 0 0])

  k       x(1)       x(2)       norm(dx)
  0     0.00000    0.00000
  0     0.00000    1     -4.00000    -2.00000 -3.000000000000000
5.385165e+000     2    -26.00000    -4.00000 -17.000000000000000
2.615339e+001     3    -156.00000    34.00000 -97.000000000000000
1.573023e+002     4    -974.00000   552.00000 -653.000000000000000
1.116505e+003     5    -6400.00000   5590.00000 -5025.000000000000000
8.598680e+003     6    -43994.00000   49988.00000 -41333.000000000000000
6.857669e+004     7    -313956.00000   426586.00000 -347977.000000000000000
5.556402e+005     8    -2310326.00000   3569424.00000 -2941037.000000000000000
4.537277e+006     9    -17431384.00000   29608942.00000 -24785073.000000000000000
3.720033e+007    10    -134197250.00000   244655180.00000 -207928325.000000000000000
3.056478e+008    11    -1049838684.00000   2017984978.00000
-1736880313.000000000000000
2.514119e+009    12    -8317017086.00000   16631509176.00000
-14457251837.000000000000000
2.069239e+010    13    -66533611696.00000   137020575814.00000
-120006529377.000000000000000
1.703624e+011    14    -536222245994.00000   1128670125332.00000
-994080327893.000000000000000
1.402848e+012    15    -4346003601300.00000   9296403653770.00000
-8221822218025.000000000000000
1.155277e+013    16    -35372425261574.00000   76568069959968.00000
-67923740615693.000000000000000
9.514429e+013    17    -288802621529416.00000   630628326252766.00000
-560678583614097.000000000000000
7.835923e+014    18    -2363444055429266.00000   5193929746274204.00000
-4625328154190117.000000000000000
6.453622e+015    19    -19374594078849800.00000   42777678841454720.00000
-3813982841768911O.000000000000000
```

```
5.315204e+016     20    -1590252433145535OO.00000  352320323962956400.00000
-314394328189044100.000000000000000
4.377619e+017
ans =

  1.0e+017 *

   -1.5903    3.5232   -3.1439

seidel('gee4',[0 0 0])

  k        x(1)        x(2)        norm(dx)
  0     0.00000     0.00000
  0     0.00000     1    -30.00000    14.00000 -7.000000000000000
2.863564e+001      2    -1486.00000  1974.00000 -633.000000000000000
2.116919e+003      3    -89370.00000  156170.00000 -51805.000000000000000
1.537103e+005      4    -5887390.00000  11194134.00000 -3776877.000000000000000
1.079355e+007      5    -400347906.00000  781494434.00000 -265135333.000000000000000
7.490895e+008      6    -27504310390.00000  54130767390.00000
-18396450285.000000000000000
5.179327e+010      7    -1895673913050.00000  3740354338154.00000
-1271846970997.000000000000000
3.576858e+012      8    -130786604686126.00000  258259398482694.00000
-87831517400733.000000000000000
2.469281e+014      9    -9026072611893810.00000  17827832775183410.00000
-6063386008998565.000000000000000
1.704472e+016     10    -622983647794921600.00000  1230579263158407000.00000
-418536390285279200.000000000000000
1.176505e+018     11    -42999916409016550000.00000  84939730987192110000.00000
-28889277360609540000.000000000000000
8.120685e+019     12    -2967991546733411000000.00000  5862854804676491000000.00000
-1994048004334984000000.000000000000000
5.605189e+021     13    -204860853432267700000000.00000
404675073624582200000000.00000 ****************************************
3.868899e+023     14    -14140204493014060000000000.00000
27932092852186980000000000.00000 ****************************************
2.670449e+025     15    -976006085246276400000000000.00000
1927970607579416000000000000.00000 ****************************************
1.843237e+027     16    -67367339881182130000000000000.00000
133075256522401400000000000000.00000 ****************************************
1.272267e+029     17    -4649928596882063000000000000000.00000
9185318215041278000000000000000.00000 ****************************************
8.781627e+030     18    ****************************************
634002686789934600000000000000000.00000 ****************************************
6.061385e+032     19    ****************************************
**************************************** ****************************************
4.183780e+034     20    ****************************************
**************************************** ****************************************
2.887790e+036
ans =

  1.0e+036 *

   -1.5291    3.0205   -1.0273

newtonsys('gee',[0 -3 1]',.0001,.0001,10,1)

Newton iterations
  k      norm(f)        norm(dx)
  1    2.064e+001    4.123e+000
  2    3.553e-015    9.730e-016
```

```
ans =

     1
     1
     1

diary off
```

```
function [J,f] = gee(x)
%
%
%
f(1) = 5*x(1)-x(2)-4;
f(2) = -x(1)+5*x(2)-2*x(3)-2;
f(3) = -2*x(2)+5*x(3)-3;
J = [5 -1 0; -1 5 -2 ; 0 -2 5];
f = [f(1) ; f(2) ; f(3) ];
```

```
a

a =

     5    -1     0
    -1     5    -2
     0    -2     5

b

b =

     4
     2
     3

tj
??? Undefined function or variable tj.

TJ

TJ =

         0    0.2000         0
    0.2000         0    0.4000
         0    0.4000         0

eig(TJ)

ans =

    0.0000
    0.4472
   -0.4472

norm(ans,Inf)

ans =

    0.4472

ans*ans

ans =

    0.2000

w = 2/(1+sqrt(1-0.2))

w =

    1.0557

jacobi(a,b,[0 0 0]',.00000001)
```

k	x(1)	x(2)	x(3)	norm(dx)
0	0.00000	0.00000	0.00000	
1	0.00000	0.00000	0.00000	1.077032961426901
2	0.80000	0.40000	0.60000	0.438178046004133
3	0.88000	0.80000	0.76000	0.195959179422654
4	0.96000	0.88000	0.92000	0.087635609200826
5	0.97600	0.96000	0.95200	0.039191835884531
6	0.99200	0.97600	0.98400	0.017527121840165
7	0.99520	0.99200	0.99040	0.007838367176906
8	0.99840	0.99520	0.99680	0.003505424368033
9	0.99904	0.99840	0.99808	0.001567673435381
10	0.99968	0.99904	0.99936	0.000701084873606
11	0.99981	0.99968	0.99962	0.000313534687076
12	0.99994	0.99981	0.99987	0.000140216974721
13	0.99996	0.99994	0.99992	0.000062706937415
14	0.99999	0.99996	0.99997	0.000028043394944
15	0.99999	0.99999	0.99998	0.000012541387483
16	1.00000	0.99999	0.99999	0.000005608678989
17	1.00000	1.00000	1.00000	0.000002508277497
18	1.00000	1.00000	1.00000	0.000001121735798
19	1.00000	1.00000	1.00000	0.000000501655499
20	1.00000	1.00000	1.00000	0.000000224347160

```
ans =

   1.0000    1.0000    1.0000

gseidel(a,b,[0 0 0]',.00000001)

Using Gauss-Seidel iteration:
```

k	x(1)	x(2)	x(3)	norm(dx)
0	0.00000	0.00000	0.00000	
1	0.00000	0.00000	0.00000	1.277722974670175
2	0.80000	0.56000	0.82400	0.395313344070245
3	0.91200	0.91200	0.96480	0.103466446735161
4	0.98240	0.98240	0.99296	0.020693289347032
5	0.99648	0.99648	0.99859	0.004138657869406
6	0.99930	0.99930	0.99972	0.000827731573881
7	0.99986	0.99986	0.99994	0.000165546314776
8	0.99997	0.99997	0.99999	0.000033109262955
9	0.99999	0.99999	1.00000	0.000006621852591
10	1.00000	1.00000	1.00000	0.000001324370518
11	1.00000	1.00000	1.00000	0.000000264874104
12	1.00000	1.00000	1.00000	0.000000052974821
13	1.00000	1.00000	1.00000	0.000000010594964
14	1.00000	1.00000	1.00000	0.000000002118993

```
ans =

   1.0000    1.0000    1.0000

sor(a,b,[0 0 0]',1,.00000001)

??? sor(a,b,[
            |
Undefined function or improper matrix reference.

sor(a,b,[0 0 0],1,.00000001)

??? sor(a,b,[
            |
Undefined function or improper matrix reference.
```

```
pwd

ans =

Q:\MFILES\MATH370\NMM\ROOTFIND

cd ../linalg

sor(a,b,[0 0 0],1,.00000001)

Using Successive Over-Relaxation (SOR) Method:
  k       x(1)        x(2)         x(3)       norm(dx)
  0     0.00000     0.00000      0.00000
??? Error using ==> *
Inner matrix dimensions must agree.

Error in ==> q:\mfiles\math370\nmm\linalg\sor.m
On line 33  ==>       f(1) = (b(1)-matrix(1,2:numcomps)*x(2:numcomps))/matrix(1,1);

sor(a,b,[0 0 0]',1,.00000001)

Using Successive Over-Relaxation (SOR) Method:
  k       x(1)        x(2)         x(3)       norm(dx)
  0     0.00000     0.00000      0.00000
  1     0.00000     0.00000      0.00000    1.277722974670175
  2     0.80000     0.56000      0.82400    0.395313344070245
  3     0.91200     0.91200      0.96480    0.103466446735161
  4     0.98240     0.98240      0.99296    0.020693289347032
  5     0.99648     0.99648      0.99859    0.004138657869406
  6     0.99930     0.99930      0.99972    0.000827731573881
  7     0.99986     0.99986      0.99994    0.000165546314776
  8     0.99997     0.99997      0.99999    0.000033109262955
  9     0.99999     0.99999      1.00000    0.000006621852591
 10     1.00000     1.00000      1.00000    0.000001324370518
 11     1.00000     1.00000      1.00000    0.000000264874104
 12     1.00000     1.00000      1.00000    0.000000052974821
 13     1.00000     1.00000      1.00000    0.000000010594964
 14     1.00000     1.00000      1.00000    0.000000002118993

ans =

    1.0000    1.0000    1.0000

sor(a,b,[0 0 0]',1.05573,.00000001)

Using Successive Over-Relaxation (SOR) Method:
  k       x(1)        x(2)         x(3)       norm(dx)
  0     0.00000     0.00000      0.00000
  1     0.00000     0.00000      0.00000    1.364175044273956
  2     0.84458     0.60062      0.88708    0.380595585435475
  3     0.92433     0.95859      0.98881    0.081334116410465
  4     0.99547     0.99663      0.99920    0.005182335175282
  5     0.99954     0.99975      0.99994    0.000494244369941
  6     0.99997     0.99998      1.00000    0.000029447716680
  7     1.00000     1.00000      1.00000    0.000002284946685
  8     1.00000     1.00000      1.00000    0.000000133098595
  9     1.00000     1.00000      1.00000    0.000000009422569

ans =

    1.0000    1.0000    1.0000
```

sor(a,b,[0 0 0]',1.1,.00000001)

Using Successive Over-Relaxation (SOR) Method:

k	x(1)	x(2)	x(3)	norm(dx)
0	0.00000	0.00000	0.00000	
1	0.00000	0.00000	0.00000	1.434281826788585
2	0.88000	0.63360	0.93878	0.370392864334364
3	0.93139	0.99461	1.00375	0.074851437458822
4	1.00568	1.00344	1.00114	0.006476913373018
5	1.00019	1.00020	0.99997	0.000278450113703
6	1.00002	0.99997	0.99999	0.000041022683551
7	0.99999	1.00000	1.00000	0.000008962597950
8	1.00000	1.00000	1.00000	0.000000188699128
9	1.00000	1.00000	1.00000	0.000000037413742
10	1.00000	1.00000	1.00000	0.000000007167307

ans =

    1.0000   1.0000   1.0000

sor(a,b,[0 0 0]',1.8,.00000001)

Using Successive Over-Relaxation (SOR) Method:

k	x(1)	x(2)	x(3)	norm(dx)
0	0.00000	0.00000	0.00000	
1	0.00000	0.00000	0.00000	2.737632260896997
2	1.44000	1.23840	1.97165	1.623655021930376
3	0.73382	1.41304	0.52007	1.239056275390026
4	1.36164	0.45421	0.99097	1.183138815822655
5	0.51421	1.25525	1.19100	1.013598387042754
6	1.48052	1.10631	0.92374	0.907618664534319
7	0.65385	0.73543	0.87052	0.783064863042409
8	1.18167	1.18383	1.23594	0.536477525857208
9	0.92084	0.99432	0.80715	0.319373222945134
10	1.06128	0.88776	1.07346	0.274043635217799
11	0.91057	1.11049	1.02078	0.254492583037132
12	1.11132	0.96665	0.95936	0.217196207423569
13	0.89893	0.96104	1.00446	0.197062165413096
14	1.06683	1.05844	1.03851	0.161716072004115
15	0.96758	0.96930	0.94709	0.103577563702743
16	1.01489	0.99182	1.03644	0.065220910346689
17	0.98515	1.02743	0.99060	0.060645688743969
18	1.02176	0.97912	0.99249	0.054299310662889
19	0.97508	1.00232	1.00768	0.046903115190615
20	1.02077	1.01115	1.00188	0.042444845014145

ans =

    0.9874   0.9879   0.9898

sor(a,b,[0 0 0]',0.9,.00000001)

Using Successive Over-Relaxation (SOR) Method:

k	x(1)	x(2)	x(3)	norm(dx)
0	0.00000	0.00000	0.00000	
1	0.00000	0.00000	0.00000	1.127444374475300
2	0.72000	0.48960	0.71626	0.418724982147101
3	0.88013	0.82524	0.90871	0.152268918133477
4	0.95656	0.94184	0.96993	0.052285926432190
5	0.98519	0.98069	0.99004	0.017545178233258
6	0.99504	0.99359	0.99670	0.005842581241082
7	0.99835	0.99787	0.99890	0.001940971676828
8	0.99945	0.99929	0.99964	0.000644346843680

```
  9     0.99982     0.99977      0.99988  0.000213857931589
 10     0.99994     0.99992      0.99996  0.000070974511893
 11     0.99998     0.99997      0.99999  0.000023554334899
 12     0.99999     0.99999      1.00000  0.000007816938205
 13     1.00000     1.00000      1.00000  0.000002594189690
 14     1.00000     1.00000      1.00000  0.000000860927427
 15     1.00000     1.00000      1.00000  0.000000285713846
 16     1.00000     1.00000      1.00000  0.000000094819139
 17     1.00000     1.00000      1.00000  0.000000031467390
 18     1.00000     1.00000      1.00000  0.000000010443004
 19     1.00000     1.00000      1.00000  0.000000003465694

ans =

    1.0000    1.0000    1.0000

diary off
```

```
type hw33

function [f,dfdx] = hw33(x)
f = x.^2 + 1/(x.^2) -2;
dfdx = 2*x -2/(x.^3);
newton('hw33',.5,.0001,.0001,1)


Newton iterations for hw33.m
  k      f(x)          dfdx           x(k+1)
  1   2.250e+000   -1.500e+001    0.65000000000000
  2   7.894e-001   -5.983e+000    0.78194200351494
  3   2.469e-001   -2.619e+000    0.87621727601415
  4   7.025e-002   -1.221e+000    0.93377485083229
  5   1.881e-002   -5.889e-001    0.96571597200662
  6   4.870e-003   -2.892e-001    0.98255389045529
  7   1.239e-003   -1.433e-001    0.99119951446113
  8   3.126e-004   -7.135e-002    0.99558022394818
  9   7.848e-005   -3.559e-002    0.99778520673686

ans =

    0.9978

newton('hw33',-.5,.0001,.0001,1)


Newton iterations for hw33.m
  k      f(x)          dfdx           x(k+1)
  1   2.250e+000    1.500e+001   -0.65000000000000
  2   7.894e-001    5.983e+000   -0.78194200351494
  3   2.469e-001    2.619e+000   -0.87621727601415
  4   7.025e-002    1.221e+000   -0.93377485083229
  5   1.881e-002    5.889e-001   -0.96571597200662
  6   4.870e-003    2.892e-001   -0.98255389045529
  7   1.239e-003    1.433e-001   -0.99119951446113
  8   3.126e-004    7.135e-002   -0.99558022394818
  9   7.848e-005    3.559e-002   -0.99778520673686

ans =

   -0.9978

diary off
```