Numerical Analysis

Homework Set 2

9 questions+journal+bonus, 50+10 bonus points

ASSIGNED: Mon Feb 23 2009 **DUE: Fri Mar 6 2009**

1. (*5 points*) Let $p_n = \dfrac{(a_n + b_n)}{2}, r = \lim_{n\to\infty} p_n$ and $e_n = r - p_n$. In this problem $[a_n, b_n]$ with $n \geq 0$ denotes the successive intervals that arise when using bisection method on a continuous function $f(x)$ with initial interval $[a, b]$.

   (a.) Show that $|e_n| \leq 2^{-n}(b - a)$

   (b.) Show that $|r - p_n| \leq |p_n - p_{n-1}|$

   (c.) Show that $|e_n| = 0 + \mathcal{O}(2^{-n})$

   (d.) Discuss whether the limit $\lim_{n\to\infty} \dfrac{|r - p_{n+1}|}{|r - p_n|}$ exists or not. If it exists, find the limit.

2. (*4 points*) Find an approximation to $\sqrt[3]{25}$ to within 10^{-4} using the Bisection Algorithm.

3. (*5 points*) Let $f(x) = x^2 - 6$. With $p_0 = 3$ and $p_1 = 2$ Find p_3 by hand using **(a.)** Secant **(b.)** False Position **(c.)** Bisection **(d.)** Newton-Raphson **(e.)** Which method produces the value of p_3 closest to the exact value of the root?

4. (*4 points*) Use Newton's Method to approximate, to within 10^{-4}, the value of x that produces the the point on the graph of $y = x^2$ that is closest to $(1, 0)$. [*HINT: Minimize* $[d((x)]^2$, *where* $d(x)$ *represents the distance from* (x, x^2) *to* $(1, 0)$.]

5. (*4 points*) The sum of two numbers is 20. If each number is added to its square root, the product of the sums is 155.55. Determine the two numbers to within 10^{-4}.

6. (*4 points*) Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$

   (a) $g_1(x) = (3 + x - 2x^2)^{1/4}$ **(b)** $g_2 = \left(\dfrac{x + 3 - x^4}{2}\right)^{1/2}$

   (c) $g_3(x) = \left(\dfrac{x + 3}{x^2 + 2}\right)^{1/2}$ **(d)** $g_4(x) = \dfrac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$

7. (*4 points*)

   (a.) Perform four iterations, if possible, on each of the functions defined in the previous question. Let $p_0 = 1$ and $p_{n+1} = g(p_n)$ for $n = 0, 1, 2, 3$

   (b.) Which function do you think gives the best approximation to the solution (and **why**)?

8. (*5 points*) **Recktenwald p. 287, #21.** David Peters (*SIAM Review*, Vol. 39, no. 1, pp. 118-122, March 1997) obtains the following equation for the optimum damping ratio of a spring-mass-damper system designed to minimize the transmitted force when an impact is applied to the mass:

$$\cos(4x\sqrt{1-x^2}) = -1 + 8x^2 - 8x^4$$

Write a MATLAB m-file which is necessary to obtain the value of x which satisfies the equation. What is the value of x which solves the equation? (HINT: You could edit and modify one of the root-finding MATLAB files we already have available to solve this problem.)

9. (*5 points*) Let A be a given positive constant and $g(x) = 2x - Ax^2$.

   (a.) Show that if fixed point iteration converges to a non-zero limit, then the limit is $p = 1/A$, so the inverse of a number can be found by only using multiplications and subtractions.

   (b.) Find an interval around $1/A$ for which fixed-point iteration converges, provided p_0 is in that interval.

JOURNAL ENTRY

(*10 points*) Use a separate sheet of paper to discuss your understanding of the different root-finding techniques you have been introduced to in this unit. Which do you think is the most useful and which do you think is the most useless? Talk about how you would use MATLAB to implement one of these root-finding techniques. How would you go about solving $f(x) = 0$ in the "real world"?
Self-Assessment: In addition, write a paragraph describing your opinion on the pacing of the course. Have we been moving too quickly, too slowly or just about right?

BONUS

(*10 points*) Player A will shut out (win by a score of 21-0) a player B in a game of racquetball with probability

$$P = \frac{1+p}{2}\left(\frac{p}{1-p+p^2}\right)^{21},$$

where p denotes the probability A wil win any specific rally (independent of the server). Write a MATLAB m-file `squash.m` to determine the minimal value of p that will ensure that A will shut out B in at least half the matches they play. You will need to email me the file with your *m-file* and include a printout of correct output to get full credit for this bonus.

NOTES

This homework set is due by 5pm on **Friday March 6**. You are **strongly** encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work. You should indicate on your neatly-written solution manuscripts which students you collaborated with. If you encounter difficulty, you should ask questions on the online message board at `http://blackboard.oxy.edu` , or via the *Numerical Analysis* class email list at `math370-L@oxy.edu`, or come see me in my office.