HW #1 Solutions

1. (a) $(121 - 0.327) = 120$ (3 digit chopping) $= 1$

$(121 - 0.327) = 121$ (3 digit rounding) $= 2$

exact $= 120.673$ ~~#2~~ 1.673

absolute error rounding $= |120.673 - 120| = 0.673$

absolute error chopping $= |120.673 - 121| = 0.327$

relative error rounding $= \dfrac{0.673}{1\cancel{20.673}}$ $\qquad \dfrac{0.673}{1.673} = 0.4027$

relative error chopping $= \dfrac{0.327}{\cancel{120.673}}$ $\qquad \dfrac{0.327}{1.673} = 0.19546$

(b) $(121-119) - 0.327 = 2 - 0.327 = 1.673$ exact

$= 2 - 0.327 = 1.67$ rounding

$2 - 0.327 = 1.67$ chopping

absolute error $= |1.673 - 1.67| = 0.003$

relative error $= \dfrac{0.003}{1.673} = 1.793 \times 10^{-3}$

~~You should subtract numbers which are~~

2. (a) $$\frac{\sin(h) - h\cos(h)}{h} \approx \frac{h - \frac{h^3}{3!} + \frac{h^5}{5!} - \frac{h^7}{7!} + \dots - h\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \frac{h^6}{6!} + \dots\right)}{h}$$

$$\approx \frac{1}{h}\left(h^3\left(\frac{1}{2} - \frac{1}{6}\right) + h^5\left(\frac{1}{120} - \frac{1}{24}\right) + \dots\right)$$

$$\approx 0 + h^2\left(\frac{1}{3}\right) + \frac{h^4}{24}\left(-\frac{4}{5}\right) + \dots$$

$$\approx 0 + \frac{h^2}{3} - \frac{h^4}{30} + \dots$$

$$\approx 0 + \mathcal{O}(h^2)$$

(b) $$\frac{1 - e^{h^2}}{h^2} \approx \frac{1 - \left[1 + h^2 + \frac{(h^2)^2}{2!} + \frac{(h^2)^3}{3!} + \dots\right]}{h^2}$$

$$\approx \frac{-h^2 - \frac{h^4}{2} - \frac{h^6}{6} \dots}{h^2}$$

$$= -1 - \frac{1}{2}h^2 - \frac{h^4}{6} \dots$$

$$= -1 + \mathcal{O}(h^2)$$

(c) $$\frac{\tan(h)}{h} \approx \frac{h + \frac{h^3}{3} + \dots}{h} \approx 1 + \mathcal{O}(h^2)$$

(d) $$\frac{1 - \cos(h)}{h} = \frac{1 - \left(1 - \frac{h^2}{2} + \frac{h^4}{4!} - \frac{h^6}{6!} + \dots\right)}{h}$$

$$= 0 + \mathcal{O}(h)$$

```
>> format long
>> cosh(5)

ans =

  74.20994852478785

>> sinh(-2)

ans =

  -3.62686040784702

>> exp(5)+exp(-5)

ans =

    1.484198970495757e+002

>> erf(1.2)

ans =

   0.91031397822964

>> beta(1,2)

ans =

   0.50000000000000

>> beta(0.4,0.7)

ans =

   3.02653229033562
>> besselj(0,2)

ans =

   0.22389077914124

>> bessely(0,2)

ans =

   0.51037567264975
```

```
   0.01000000000000   0.03162277660168   0.10000000000000   0.31622776601684

  Column 5

   1.00000000000000

>> y=10.^[1:.5:3]

y =

   1.0e+003 *

  Columns 1 through 4

   0.01000000000000   0.03162277660168   0.10000000000000   0.31622776601684

  Column 5

   1.00000000000000

>> norm(x-y)

ans =

     0
```

3.

$$F_0 = 1$$

$$F_1 = 1$$

$$F_{n+2} = F_n + F_{n+1}$$

$$x_n = \frac{F_{n+1}}{F_n} \qquad x_{n+1} = \frac{F_{n+2}}{F_{n+1}}$$

$$\frac{F_{n+2}}{F_{n+1}} = \frac{F_n}{F_{n+1}} + 1$$

$$x_{n+1} = \frac{1}{x_n} + 1$$

$$\lim_{n\to\infty} x_n = x$$

$$\lim_{n\to\infty} x_{n+1} = x$$

$$\lim_{n\to\infty} \frac{1}{x_n} = \frac{1}{x}$$

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \frac{1}{x_n} + 1$$

$$x = \frac{1}{x} + 1$$

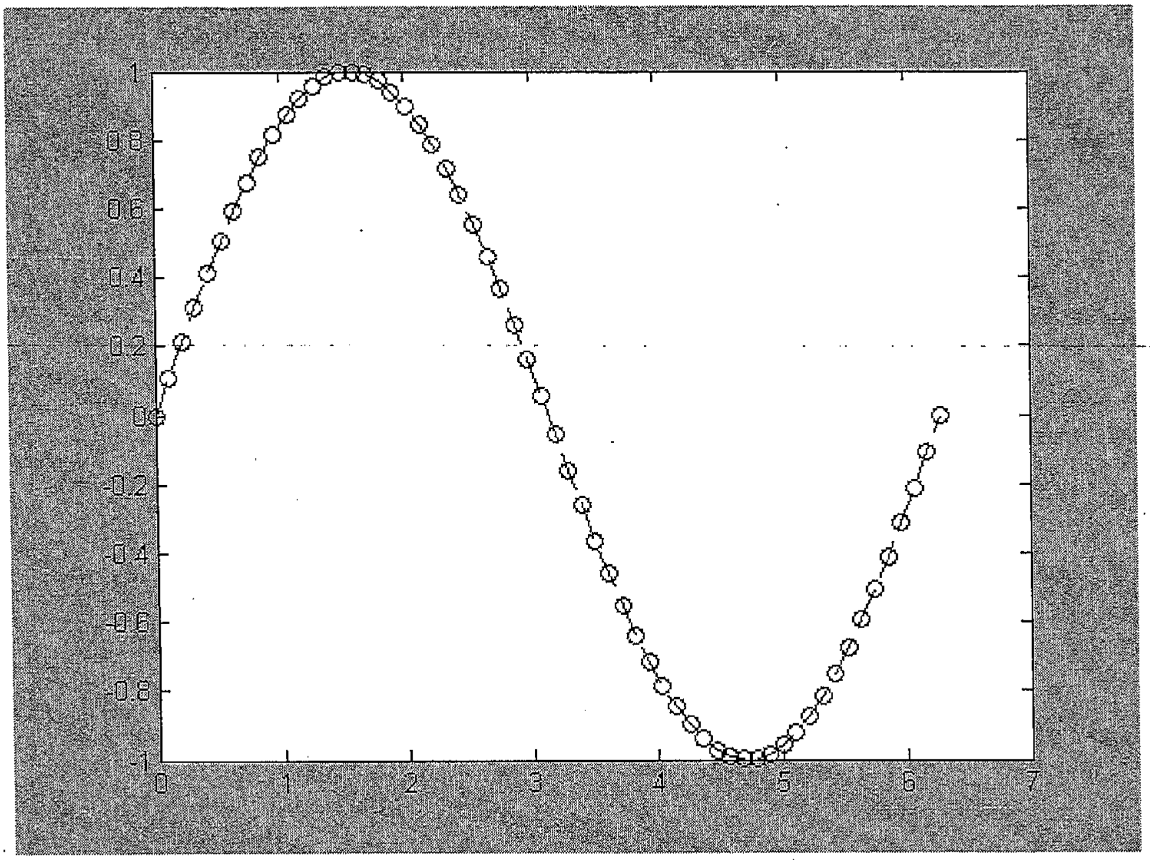
$$x^2 = 1 + x$$

$$x^2 - x - 1 = 0$$

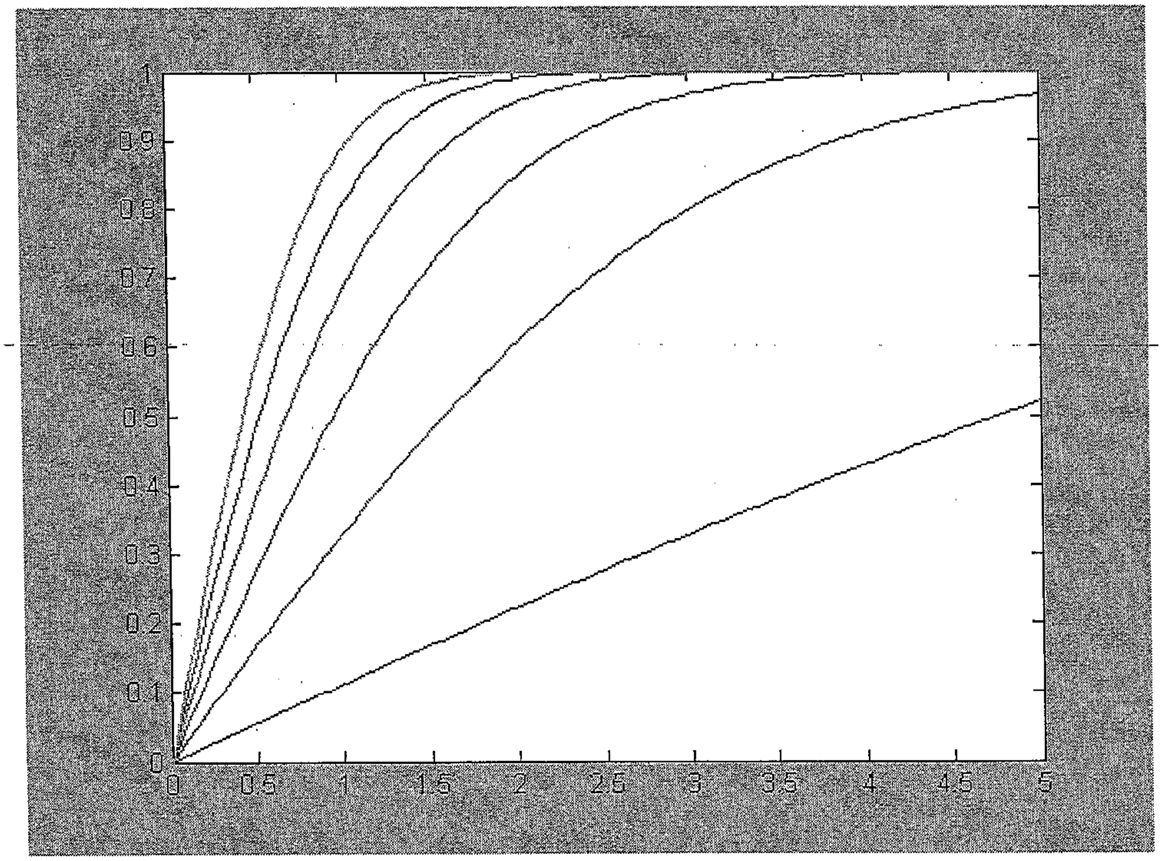
$$x = \frac{1 \pm \sqrt{1-4(-1)}}{2}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

```
>> x=linspace(0,10,5);
>>  y=[0:2.5:10];
>> norm(x-y)

ans =

   0

>> x=linspace(-5,5);
>> y=[-5:10/99:5];
>> norm(x-y)

ans =

   3.293453726225543e-015

>> eps

ans =

   2.220446049250313e-016

>> x=logspace(1,3,3)

x =

        10        100        1000

>> y=10.^[1:3]

y =

        10        100        1000

>> norm(x-y)

ans =

   0

>> x=logspace(1,3,5)

x =

  1.0e+003 *

  Columns 1 through 4
```




```
>> x=linspace(0,2*pi,60);
>> y=sin(x);
>>  plot(x,y,'o--')
```




```
>> xx=linspace(0,5,100);
yy=erf(0.1*xx);
yy2=erf(0.3*xx);
yy3=erf(.5*xx);
yy4=erf(.7*xx);
yy5=erf(.9*xx);
yy6=erf(1.1*xx);
plot(xx,yy,xx,yy2,xx,yy3,xx,yy4,xx,yy5,xx,yy6);
```

BONUS

(a) $\sum_{i=1}^{N} \sum_{j=1}^{i} a_i b_j$

$N=1$

$\sum_{i=1}^{1} \sum_{j=1}^{i} a_i b_j = a_1 b_1$ 1 MULT 0 ADDS

$N=2$

$\sum_{i=1}^{2} \sum_{j=1}^{i} a_i b_j = \sum_{j=1}^{2} a_2 b_j + \sum_{j=1}^{1} a_1 b_j$

$= a_2 b_1 + a_2 b_2 + a_1 b_1$

2 ADDS
3 MULTS

$N=3$

$\sum_{i=1}^{3} \sum_{j=1}^{i} a_i b_j = a_3 b_1 + a_3 b_2 + a_3 b_3 + a_2 b_1 + a_2 b_2 + a_1 b_1$

5 ADDS
6 MULTS

$\vdots$

N

$$\underbrace{\underbrace{[a_N b_1 + a_N b_2 + \dots + a_N b_N]}_{K=N} + \underbrace{[a_{N-1} b_1 + a_{N-1} b_2 + \dots a_{N-1} b_{N-1}]}_{K=N-1} \dots + \underbrace{[a_2 b_1 + a_2 b_2]}_{K=2} + \underbrace{a_1 b_1}_{K=1}}_{N \text{ groups}}$$

each group has K terms with K MULTS, K-1 ADDS
where K = 1 to N

To Add N groups causes N-1 ADDS

To add K MULTS from 1 to N

$\sum_{K=1}^{N} K = \frac{N(N+1)}{2}$

$\sum_{K=1}^{N} K-1 = \frac{N(N-1)}{2}$

TOTAL ADDS $= N-1 + \frac{N(N-1)}{2} = \frac{(N-1)(N+2)}{2}$

(b) $\displaystyle\sum_{i=1}^{N} \sum_{j=1}^{i} a_i b_j$ $\quad \dfrac{N(N+1)}{2}$ MULTS

$\dfrac{(N-1)(N+2)}{2}$ ADDS

$$
\begin{aligned}
= \; & a_N b_1 + a_N b_2 + a_N b_3 + \cdots a_N b_N \\
& + a_{N-1} b_1 + a_{N-1} b_2 + a_{N-1} b_3 + \cdots + a_{N-1} b_{N-1} \\
& + a_{N-2} b_1 + a_{N-2} b_2 + \cdots + a_{N-2} b_{N-2} \\
& + \\
& \vdots \\
& + a_2 b_1 + a_2 b_2 \\
& + a_1 b_1
\end{aligned}
$$

We can change this to look like:

$$
\left.
\begin{aligned}
& a_N \cdot (b_1 + b_2 + b_3 + \cdots + b_N) \\
& + a_{N-1} \cdot (b_1 + b_2 + \cdots + b_{N-1}) \\
& + a_{N-2} \cdot (b_1 + b_2 + \cdots + b_{N-2}) \\
& + \; \vdots \\
& + a_2 \cdot (b_1 + b_2) \\
& + a_1 \cdot b_1
\end{aligned}
\right\}
$$

N MULTS

Same # of ADDS $\dfrac{(N-1)(N+2)}{2}$

$$= \sum_{i=1}^{N} a_i \cdot \sum_{j=1}^{i} b_j$$