Test 2 (REPRISE): Numerical Analysis

Math 370 Spring 2009 — Friday April 22

 — DUE WED APR 29 5pm

Name: ______________________________

Directions: Read *all* problems first before answering any of them. There are 6 pages in this test. This is an official extra credit opportunity for Math 370. In order to get credit for this ECO you must get a score of 90 or higher on your re-write. This means you must complete the entire exam, not just correcting problems you had incorrect before. You may talk to other professors or myself but to no other students in this class. **Use of calculators is allowed, but unnecessary!** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work." Since this is a take-home exam I will expect very cler explanations and all details of all calculations to be shown.

Pledge: I, ______________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		40
2		30
3		30
BONUS		10
Total		**100**

1. *[40 points total.]* **Polynomial Interpolation.**

Consider the data

i	x_i	y_i
0	-1	2
1	0	1
2	2	1/2

(a) *[10 pts]* For the given data, find the second degree Newton interpolating Polynomial $N(x)$ and use it to interpolate the value $N(1)$.

(b) *[10 pts]* For the given data, find the second degree Lagrange interpolating Polynomial $L(x)$ and use it to interpolate the value $L(1)$.

i	x_i	y_i
0	-1	2
1	0	1
2	2	1/2

(c) *[5 pts]* Consider the $(n+1)^{th}$-degree Legendre polynomials defined as

$$G_{n+1}(x) = xG_n(x) - \frac{n^2}{4n^2-1}G_{n-1}(x) \text{ for } n \geq 1$$

where $G_0(x) = 1$ and $G_1(x) = x$. Obtain $G_2(x)$.

(d) *[10 pts]* Use the given data to set up a linear system to find the equation of the second degree Legendre polynomial $G(x) = c_0G_0(x) + c_1G_1(x) + c_2G_2(x)$. Find c_0, c_1 and c_2. (HINT: you should be able to answer this question without using any Linear Algebra!)

(e) *[5 pts]* What do the values $G(1)$, $(L(1)$ and $N(1)$ have in common? EXPLAIN YOUR ANSWER.

2. *[30 pts. total]* **Approximation Theory.**

Consider the following MATLAB script. Ignore the (LINE NUMBERS) they are merely for your benefit.

```
(LINE 1) x=[-1 0 2];
(LINE 2) y=[2 1 0.5];
(LINE 3) Y=1./y;
(LINE 4) c=polyfit(x,Y,1);
(LINE 5) xx=linspace(x(1),x(length(x)));
(LINE 6) yy=polyval(c,xx);
(LINE 7) plot(x,y,'kd',xx,yy,'b-')
```

(a) *[15 pts]* For each line of the script, write a short sentence explaining what that line does.

(b) *[15 pts]* **(i)** Explain what the output of the script would be and what the entire script is trying to do. **(ii)** What is the presumed relationship between the data in the variable x and the data in the variable y? **(iii)** Discuss if you think the script is successful, and if you think it is not successful, how you would modify it to produce correct output.

3. *[30 points total.]* **TRUE or FALSE.**

INSTRUCTIONS: Are the following statements TRUE or FALSE – put your answer in the box. To receive ANY credit, you must also give a brief, and correct, explanation in support of your answer! For example, if you think the answer is FALSE providing a counter example for which the statement is not true is best. If you think the answer is TRUE you should also explain why you believe the statement.

(a) TRUE OR FALSE? "An interpolated value is always more accurate than an extrapolated value."

(b) TRUE OR FALSE? "For a diagonal matrix A of real positive numbers, the matrix 1-norm equals the matrix infinity-norm, i.e. $||A||_1 = ||A||_\infty$"

(c) TRUE OR FALSE? "Given experimental data representing a presumed nonlinear relationship between the dependent variable y and the independent variable x, in order to find the curve of best fit (that minimizes the least square error) one must solve a nonlinear system of equations."

BONUS QUESTION. Spectral Radius. (*10 points.*)

Consider the diagonal matrix $A = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix}$ where $\lambda_i > 0$ for $i = 1, ..n$. Recall that $||A||_2 = \sqrt{\rho(A^T A)}$ where $\rho(A) = \max_{1 \leq i \leq n} \lambda_i(A)$ and $\lambda_i(A)$ is an eigenvalue of A. Show that $||A||_1 = ||A||_2$.