Numerical Analysis

Math 370 Fall 2002 MWF 9:30am - 10:25pm

 Fowler 127

Worksheet 26

SUMMARY Implementation of Cubic Splines
READING Recktenwald, pp. 521–538

Summary Given that $s_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + d_k(x - x_k)^3$ and that we have $N + 1$ data points (x_k, y_k) how many cubic splines will we use to interpolate the entire interval from x_0 to x_N? How many unknown constants do we have to compute (i.e find) in order to define all of these $s_k(x)$?

How many equations do we have to solve for to find these unknown constants?
Write them down.

What is a **natural spline**?

What is a **clamped spline**?

There isn't a known error formula for the **natural cubic spline**, but there is one for the clamped cubic spline:
Let M be a bound for $f^{(4)}(x)$ on $[a, b]$. Then if S is the unique clamped spline interpolant with respect to the nodes $a = x_0 < x_1 < \ldots < x_n = b$ then

$$\max_{a \leq x \leq b} |f(x) - S(x)| \leq \frac{5M}{384} \max_{0 \leq j \leq n-1} (x_{j+1} - x_j)^4$$

MATLAB implementation of cubic splines

1. Let's compute the clamped cubic spline for the data (1,1), (2,3), (3,2), (4,4). We want the derivatives at the end points to be 0. Look at the MATLAB `splint.m` and/or `splintFE.m` programs.

2. What commands would you have to use for the natural spline?

3. Write down the commands necessary to plot the cubic spline AND the data. Choose different colors for the data and the piecewise interpolants.