# Numerical Analysis

Math 370 Fall 2002 MWF 9:30am - 10:25pm

 Fowler 127

*Worksheet 16*

**SUMMARY** Iterative Methods for Solving Systems of Equations (continued)
**READING** Recktenwald, Sec 8.5, pp. 427–445

*Example*
Let's use MATLAB to compare Seidel Iteration, Picard Iteration (Successive Substitution) and Newton's Method. Consider the system

$$\begin{aligned} y &= 1.4x - 0.6 \\ y &= x^2 - 1.6x - 4.6 \end{aligned}$$

We know the system has two solutions : (-1,-2) and (4,5). Depending on the initial guess, the algorithms will converge to one or the other solution.
We can write this as a matrix equation $A\vec{x} = \vec{b}$

$$\begin{pmatrix} 1.4 & -1 \\ x - 1.6 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.6 \\ 4.6 \end{pmatrix}$$

On Wednesday we showed that one could rewrite Newton's Method for Systems to generate a Functional Iteration scheme by using Cramer's Rule on the system $J\Delta\vec{x} = -\vec{f}(\vec{x})$. Though, in practice, one doesn't usually think of the solution method that way.
Using Cramer's Rule we obtained the expression for $\vec{x} = A^{-1}\vec{b}$

$$x = \frac{4}{x-3}, \qquad y = \frac{7.4 - 0.6x}{x-3}$$

Write this down as a Successive Substitution iteration scheme $\vec{x}_{k+1} = \vec{G}(\vec{x}_k)$

Write down the corresponding Seidel Iteration scheme

**seidel.m and succsub.m**

I have implemented Seidel Iteration and Successive Substitution in MATLAB . In the m-file `linepara.m` I have implemented the system which represents the intersection of the parabola and line, above. Recall that in `demossub.m` and `demonewtonsys.m` Recktenwald has implemented Successive Substitution and Newton's Method for this particular problem.

GROUPWORK

Compare the rate of convergence for using Seidel Iteration, Newton's Method and Successive Substitution to solve the system above. How many steps does each take to get within a tolerance of $10^{-8}$? How close is each after 3 steps? Which method do you prefer?

Consider the hyperbola $4x^2 - y^2 = 1$ and the circle $(x-1)^2 + y^2 = 4$. Modify the `demonewtonsys.m` program and use an initial guess of $(1.1, 2)^T$ to find a point of intersection of the two curves. How many points of intersection do the two curves have? Can you find them all numerically?