Numerical Analysis

Math 370 Fall 2002 MWF 9:30am - 10:25pm

 Fowler 127

Worksheet 15

SUMMARY Iterative Methods for Solving Systems of Nonlinear Equations
READING Recktenwald, Sec 8.5, pp. 427–445

Example
Let's consider the equation again.

$$\begin{aligned} y &= \alpha x + \beta \\ y &= x^2 + \sigma x + \tau \end{aligned}$$

This nonlinear system consists of the equations for a line and a parabola, respectively. Our problem is to find the coordinates of the point of intersection for these two curves, for any line and parabola in this form.
Exercise
Consider the system

$$\begin{aligned} y &= 1.4x - 0.6 \\ y &= x^2 - 1.6x - 4.6 \end{aligned}$$

We know the system has two solutions : (-1,-2) and (4,5). Depending on the initial guess, the algorithms will converge to one or the other solution.
1. Use the quadratic formula to confirm the two solutions to the systems are indeed (-1,-2) and (4,5).

2. Use Cramer's Rule or some other method to write the system in the form $\underline{x}^{(k+1)} = \underline{G}(\underline{x}^{(k)})$. [This should remind you of Picard Iteration's $x_{k+1} = G(x_k)$]

$$\begin{aligned} x &= g_1(x,y) && (1) \\ y &= g_2(x,y) && (2) \end{aligned}$$

Solutions should be: $g_1(x,y) = \dfrac{4}{x-3}, \qquad g_2(x,y) = \dfrac{7.4 - 0.6x}{x-3}$

There's a reasonably obvious way to improve the successive substitution method $\underline{x}^{(k+1)} = \underline{G}(\underline{x}^k)$. HINT: are we using all the information we have as soon as we have it? This improvement is called **Seidel Iteration**.

3. Write down the Jacobian for the system and use that to find the iterative step involved in Newton's Method for this system.

4. Let's do 2 iterations by hand of each method using $\vec{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Successive Substitution Iterative Step

$$\begin{aligned} A(\underline{x}^{(k)})\Delta\underline{x} &= -\underline{f}(\underline{x}^{(k)}) \\ \underline{x}^{(k+1)} &= \underline{x}^{(k)} + \Delta\underline{x}^{(k)} \end{aligned}$$

which can also be represented as solving $A(\underline{x}^{(k)})\underline{x}^{(k+1)} = b(\underline{x}^{(k)})$. Both of these should be equivalent to using $\underline{x}^{(k+1)} = \underline{G}(\underline{x}^{(k)})$ from above.

5. Let's do 2 iterations by hand using **Seidel Iteration**. See the difference between Successive Substitution and Seidel Iteration? (Note the terms in the formula below are the *components* of the vectors $\underline{x}^{(k)}$ and $\underline{x}^{(k+1)}$.

Seidel Iterative Step

$$\begin{aligned} x_1^{(k+1)} &= G_1(x^{(k)} \\ x_2^{(k+1)} &= G_2(x_1^{(k+1)}, x_2^{(k)}, \ldots, x_n^{(k)}) \\ &\vdots \\ x_n^{(k+1)} &= G_N(x_1^{(k+1)}, x_2^{(k+1)}, x_2^{(k+1)}, \ldots, x_{n-1}^{(k+1)}, x_n^{(k)}) \end{aligned}$$

6. Do 2 iterations by hand using **Newton's Method**.
Newton's Method Iterative Step

$$
\begin{aligned}
J(\underline{x}^{(k)})\Delta\underline{x} &= -\underline{f}(\underline{x}^{(k)}) \\
\underline{x}^{(k+1)} &= \underline{x}^{(k)} + \Delta\underline{x}^{(k)}
\end{aligned}
$$

7. Do you see how the methods (Newton's, Successive Substitution, and Seidel Iteration) are similar and different? **List the differences and similarities below.**

8. Now use MATLAB programs `demossub` and `demosysnewton` and `linepara` to confirm your calculations in 4,5 and 6.