Numerical Analysis

Math 370 Fall 2002 MWF 9:30am - 10:25pm

 Fowler 127

Class 12: **Friday October 4**

SUMMARY The Newton-Raphson Method and the Secant Method
READING Recktenwald, 6.1.1 (240-250)

The Newton-Raphson Algorithm

Bisection and False Position are both **globally convergent** algorithms, because, given a bracket which contains a solution, they both will find the solution, eventually.

Newton's Method (and the Secant Method) are very different from these methods, in that instead of needing a bracket where the solution exists [i.e. continuous function has values at the bracket endpoints have opposite sign] one needs a **single** guess of the solution, which has to be "close" to the exact answer, in order for these **locally convergent** to get the solution.

A Derivation of Newton's Method

Write down the first 3 terms of a Taylor expansion of $f(x)$ about the point $(p_0, f(p_0))$

Evaluate this function at the root, the point $(p_1, 0)$ and solve for p_1

This is the iterative step for Newton's Method

$p_{n+1} =$

Pseudocode for Newton's Method

```
INPUT: x_0, f(x), f'(x)
FOR k = 1 to NSTEPS
x_{k+1} = x_k - f(x_k)/f'(x_k)
OUTPUT k, x_k, f(x_k)
IF ''CONVERGED'', STOP
END
```

Example

Consider the function $f(x) = x^2 - A$, where $A > 0$

Compute the Newton iterative step using the above function $f(x)$

Simplify it, so that it look like $x_{n+1} = \dfrac{x_n + A/x_n}{2}$. Recognize this iteration?

Exercise

Let $A = 2$ and $x_0 = 1$. Find x_3

Secant Method

The secant method is very similar to Newton's method, except that instead of actually computing the derivative, one approximates it using a difference quotient. This ends up in making the iterative step look algebraically identical to the one for the False Position method.

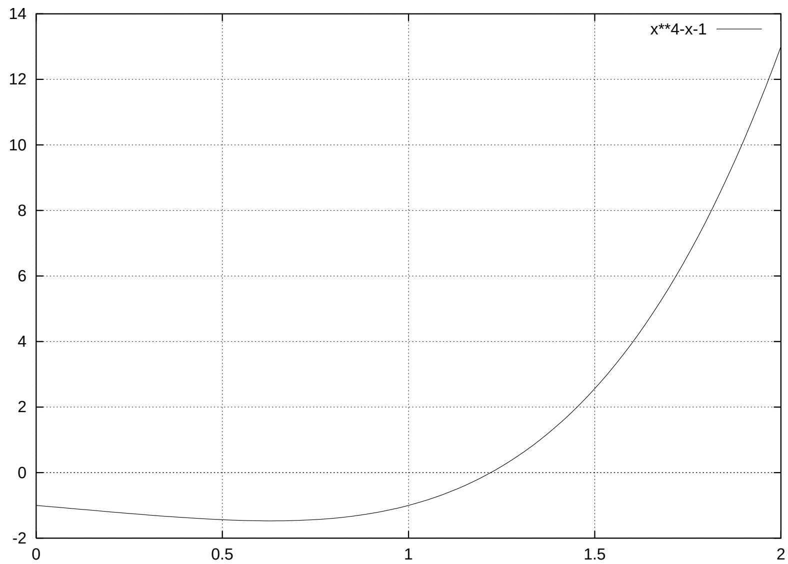
__**Exercise**__

We will write down the **Secant Method iterative step** below

$p_{n+1} =$

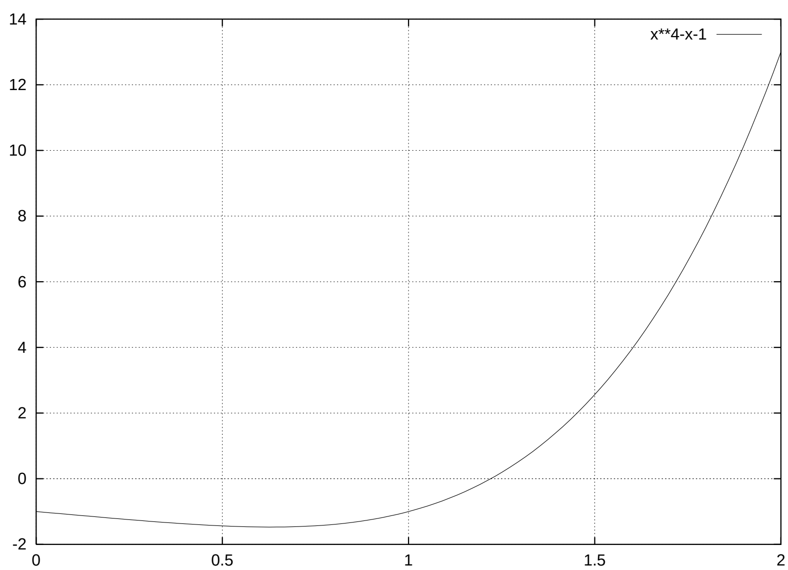
If the iterative step is identical to False Position, how come the Secant Method is not just called the False Position method? Look at the picture...

Secant Method, visually



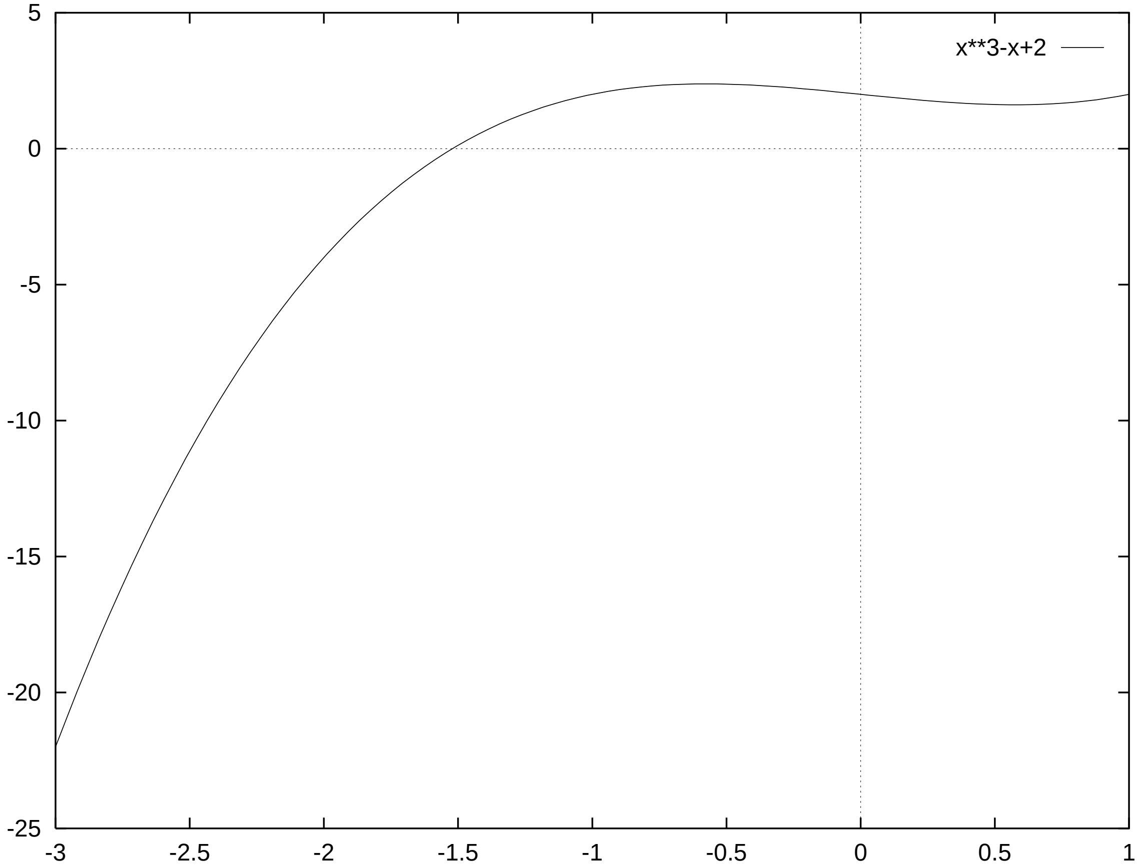
False Position Method, visually

Now, let's recall what False Position look like, visually...



Example

Consider the following function $f(x) = x^3 - x + 2$. On the first figure, draw on the graph the set of approximations to the zero, i.e. $\{p_k\}$, due to Newton's Method, if you start at $p_0 = 1$



On the following figure, draw on the graph the set of approximations produced by Bisection, Secant and False Position (use differently colored pens).