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# Numerical Analysis

Math 370 Fall 2002

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MWF 9:30am - 10:25pm

Fowler 127

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## *Worksheet 10: Monday September 30*

**SUMMARY** Introduction to Root Finding

**READING** Recktenwald, 6.1.1 (240-250)

### **Example**

Consider a ball constructed of wood which has a density of  $\rho = 0.638$  grams per cubic cm and the radius is  $r = 10$  cm. How much of the ball will be submerged when it is in water (with unit density)? Let  $x$  be the current depth of the sphere. The radius of the amount of the spherical section under water is obtained using Pythagoras' theorem with  $r - x$  and  $r$

$$M_w = \text{Mass of water displaced} = 1 \cdot \int_0^d \pi(r^2 - (r - x)^2) dx$$

$$M_b = \text{Mass of ball} = 4\pi r^3 \rho / 3$$

What's the equation which must be solved to find  $d$ , the distance below the surface the ball will float? (Produce an equation for  $d$  of the form  $f(d) = 0$  with  $d$  being the only letter present.)

### **Question**

How would you solve this equation for  $d$ ?

### **Root-Finding**

We will be looking at algorithms for the solution of equations of one variable, i.e. equations of the form  $f(x) = 0$ . This is often referred to as finding the **roots** of the equation  $f(x) = 0$  or finding the **zeroes** of the function  $f(x)$ .

### **Bracketing The Root**

How do we know where the roots of a function  $f(x)$  are? How can we “bracket” a zero of  $f(x)$ ?

## GROUPWORK

The MATLAB function **brackplo** will do this for us. Go to the computers and run **brackplo** on the function you need to find zeroes of to find  $d$ . I have made a function called **sphere.m** which you can use to help you. What do you see? How many roots are there? What range did you ask **brackplo** to search on?

## **The Bisection Method of Bolzano**

The bisection algorithm produces a sequence of approximations  $\{p_n\}$  to the zero of the function  $f(x)$

where  $p_n = a_n + \frac{b_n - a_n}{2} = \frac{a_n + b_n}{2}$  and the  $n$ -th bracket is described by  $[a_n, b_n]$

Write down the Bisection Algorithm in pseudocode here:

## **bisect.m**

In the NMM Toolbox, we have an implementation of the bisection algorithm in **bisect.m**. Use MATLAB to find the value of  $d$  which we have been looking for which tells us how much of the pine sphere is submerged.

$d =$

# General Root-Finding Algorithm

1. Plot the function, in order to get an initial guess for the root and to check for problems
2. Select an initial guess [or bracket ]
3. Iteratively refine your initial guess
4. Decide you are "converged" (If NO, Go To 3.)
5. Stop

## **demobisect.m**

There is another implementation of Bisection Algorithm in **r:/mfiles/math370/demobise.m** or **s:/math courses/math370/fall2000/** .

Modify this m-file to find the root of  $f(d) = 2552 - 30d^2 + d^3$

How many steps does it take to converge? Using what initial bracket?

## **Analyzing Convergence of Bisection**

Write down an expression for the size of  $|b_n - a_n|$  which depends on  $b - a$  and the  $n$ -th iterate (note:  $|b_0 - a_0| = b - a$ )

Solve this formula for  $n$ .

Try and predict how many iterations it will take Bisection to find the zero of  $f(x) = \log(x) - 5 + x$  on the interval  $[1, 9]$  to 5 decimal places

Go to the computer and see how many iterations **demobisect.m** actually takes to converge. Explain.

## Convergence Criteria

There are a number of different ways to consider that a method has “converged”

There is convergence criteria on  $f(x)$  and convergence criteria on  $x$

### Question

There is also relative convergence versus absolute convergence. Which do you think is the “best” method of assessing convergence?