Math 370 Fall 2002

Quiz **6** **DUE: MON OCT 28**

Name: _____

Date: _____ **Friday October 25**
Time Begun: _____ Ron Buckmire
Time Ended: _____

Topic : Solving Non-linear Systems of Equations

The idea behind this quiz is for you to obtain more practice solving a non-linear system of equations. Specifically, I want you to show that you can calculate using Successive Substitution, Seidel Iteration or Newton's Method.

Instructions:

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.
2. You may use the textbook or any of your class notes. You must work alone. However, you are strongly encouraged to ask questions on the Class Web Messageboard.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday October 28**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 370 Fall 2002 **SHOW YOUR WORK** Quiz **SIX**

1. In class we found one of the points of intersection of the hyperbola $4x^2 - y^2 = 1$ and the circle $(x-1)^2 + y^2 = 2^2$ to be (1.1165151,1.9966032). Let

$$\vec{G}(\vec{x}) = \begin{pmatrix} g_1(x,y) \\ \\ g_2(x,y) \end{pmatrix} = \begin{pmatrix} \dfrac{8x - 4x^2 + y^2 + 1}{8} \\ \\ \dfrac{2x - x^2 + 4y - y^2 + 3}{4} \end{pmatrix}$$

(a) *[2 pts]* Show that the fixed point(s) of the vector function $\vec{G}(\vec{x})$ are exactly the points of intersection of the hyperbola $4x^2 - y^2 = 1$ and circle $(x-1)^2 + y^2 = 4$. (HINT: one way to do this is to show algebraically that the fixed points of $\vec{G}$ satisfy the exact same equation that the points of intersection do.)

(b) *[2 pts]* Starting with an initial guess of $\vec{x}_0 = (1,2)^T$ compute the next approximation to the fixed point of $\vec{G}$ using Successive Substitution, $\vec{x}_k = \vec{G}(\vec{x}_{k-1})$

(c) *[2 pts]* Starting with an initial guess of $\vec{x}_0 = (1,2)^T$ compute the next approximation to the fixed point of $\vec{G}$ using Seidel Iteration.

(d) *[2 pts]* Considering $\vec{f}(\vec{x}) = \begin{bmatrix} 4x^2 - y^2 - 1 \\ (x-1)^2 + y^2 - 2^2 \end{bmatrix}$ Find the Jacobian matrix $J(x,y)$ for the system.

(e) *[2 pts]* Starting with an initial guess of $\vec{x}_0 = (1,2)^T$ compute the next approximation to the fixed point of $\vec{G}$ using Newton's Method.

You may have to attach/staple an extra sheet with your calculations on it to support your answers.