

Quiz 4

Numerical Analysis

Name: _____

Date: _____

Time Begun: _____

Time Ended: _____

Friday September 27

Ron Buckmire

Topic : Appreciating Quadratic Convergence

The idea behind this quiz is for you to hone your numerical intuition about quadratic convergence of a sequence.

Instructions:

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday September 30**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the following sequences $p_{n+1} = 0.5p_n$ and $q_{n+1} = 0.5q_n^2$, which are linearly convergent and quadratically convergent, respectively.
 - (a) *[4 pts]* Show that one can write expressions for the sequences in terms of the index n ; that is,
$$p_n = \frac{1}{2^n} \text{ and } q_n = \frac{1}{2^{2^n-1}}$$
 - (b) *[4 pts]* Derive expressions which show how many steps it takes for each sequence to be within ϵ of their limit $\lim_{n \rightarrow \infty} q_n = \lim_{n \rightarrow \infty} p_n = 0$
 - (c) *[2 pts]* Use your answer in part (b) to show how many steps it takes to reach a tolerance of 9 decimal places ($\epsilon = 5 \times 10^{-10}$).