Numerical Analysis

Math 370 Fall 2002 MWF 9:30am - 10:25pm

 Fowler 127

Homework Set 1

8 questions, 40 points

ASSIGNED: Wed Sep 18 2002 **DUE: Wed Sep 25 2002**

1. (*2 points*) Use three-digit rounding **and 3 digit chopping** to perform the following calculations. Compute the absolute error and relative error with the exact value determined to at least five digits. **(a)** $(121 - 0.327) - 119$ **(b)** $(121 - 119) - 0.327$

2. (*8 points*) Find the limits and the rates of convergence of the following functions to those limit in terms of $\mathcal{O}$ as $h \to 0$
   (a) $\dfrac{\sin(h) - h\cos(h)}{h}$ **(b)** $\dfrac{1 - e^{h^2}}{h^2}$ **(c)** $\dfrac{\tan(h)}{h}$ **(d)** $\dfrac{1 - \cos(h)}{h}$

3. (*3 points*) The sequence $\{F_n\}$ described by $F_0 = 1$, $F_1 = 1$, and $F_{n+2} = F_n + F_{n+1}$, if $n \geq 0$ is called the *Fibonacci sequence*. Consider the sequence $\{x_n\}$, where $x_n = F_{n+1}/F_n$. Assuming that the limit $\lim_{n\to\infty} x_n = x$ exists, show that the limit of the ratio of consecutive terms of the Fibonacci sequence is $x = (1 + \sqrt{5})/2$. This number is called the *golden ratio*.

4. (*4 points*) *Recktenwald*, **#2, page 77.** Evaluate the following quantities by using built-in MATLAB functions:
   (a) $\cosh(5)$ **(b)** $\sinh(-2)$ **(c)** $(e^5 + e^{-5})/2$ **(d)** $\mathrm{erf}(1.2)$ **(e)** $\beta(1, 2)$ **(f)** $\beta(0.4, 0.7)$
   (g) $J_0(2)$ **(h)** $Y_0(2)$

5. (*4 points*) *Recktenwald*, **#3, page 77.** Use colon notation to create vectors identical to those produced by the following MATLAB commands. Use the `norm` command to show that the vectors are identical *without* printing the elements.
   (a) `x = linspace(0,10,5)` **(b)** `x = linspace(-5,5)` **(c)** `x = logspace(1,3,3)`
   (d) `x = logspace(1,3,5)`

6. (*3 points*) *Recktenwald*, **#24, page 81.** Plot $\sin\theta$ versus θ for 60 points in the interval $0 \leq \theta \leq 2\pi$. Connect the point with a dashed line *and* label the points with open circles.

7. (*6 points*) *Recktenwald*, **#27, page 81.** Write the MATLAB statements to create a plot of $y = \mathrm{erf}(\alpha x)$ for $0 \leq x \leq 5$ and $\alpha = 0.1, 0.3, 0.5, 0.7, 0.9, 1.1$. Arrange the plot so that x is on the horizontal axis, and different curves correspond to α values. Choose 100 x values so the curves look smooth.

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8. (*10 points*) Use a separate sheet of paper to discuss your understanding of how the fact that computers have fixed amount of memory to represent floating point numbers causes different kinds of errors. In particular, give *your own* understanding of the terms **overflow**, **round-off error** and **mantissa**.

Self-Assessment: In addition, write atleast one paragraph describing how you approached this homework set, and what you found most challenging, and least challenging about it. You can also use this space to give me any other feedback on the course (quizzes, lectures, grading) that you wish.

BONUS (10 points)

(a.) How many multiplications and additions are required to determine a sum of the form $\sum_{i=1}^{n}\sum_{j=1}^{i} a_i b_j$?

(b.) Modify the sum in part **(a)** to an equivalent form that reduces the number of computations.

(c.) Write a function m-file which does both forms of the sum and outputs the number of operations for each form of the sum.

NOTES

This homework sets is due in class on **Wednesday September 25**. You are **strongly** encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work. You should indicate on your neatly-written solution manuscripts which students you collaborated with. If you encounter difficulty, you should ask questions on the online message board at `http://blackboard.oxy.edu` , or via the *Numerical Analysis* class email list at `math370-L@oxy.edu`, or come see me in my office.