Report on Test 1

Ron Buckmire

Point Distribution (N=13)

Range	92+	88+	82+	79+	74+	70+	68+	62+	60-
Grade	A	A-	B+	B	B-	C+	C	C-	F
Frequency	4	2	0	3	1	1	0	1	1

Comments

Overall Overall performance was "okay." The average was a 79.6! Almost half the class got an A on the exam. There was a perfect score of 100 and one almost-perfect score of 99.

#1 Order Notation and Taylor Series. This is a typical example of how Taylor Series approximations are really used to obtain information about unknown functions. In this case the function in question is $F(h) = \int_0^h sin(x^2)dx$. Of course, since $sin\square \approx \square - \frac{\square^3}{3!} + \frac{\square^5}{5!} - \ldots$ we know that $\sin(x^2) \approx x^2 - x^6/6 + x^{10}/120$. We can use this information to get an approximation for the functional behavior of $F(h)$ by integrating the Taylor series approximation of its unintegrable integrand function, $\sin(x^2)$. This gets us the results that $F(h) \approx \frac{h^3}{3} - \frac{h^7}{42} + \frac{h^11}{1320}$ but all we really care about is the first two terms, i.e. $0 + \mathcal{O}(h^3)$. The last part is to use L'Hopital's Rule to confirm our result. This forces us to compute $F'(h)$ which is simply $sin(h^2)$.

#2 Sequences and Limits. This problem is really about getting to the heart of your understanding of sequences and recursion. The first problem is just for you to get a sense of what the sequence we're dealing with is, which is $x_{n+1} = \sqrt{a + x_n}$ with $x_0 = \sqrt{a}$. So $x_\infty = \sqrt{a + \sqrt{a + \sqrt{a + \ldots}}}$ $\lim_{n\to\infty} x_n$ must equal $\lim_{n\to\infty} x_{n+1}$ and they both are equal to x. So basically where one has x_n or x_{n+1} one can replace it with x. This produces the equation: $x = \sqrt{a + x}$ which is $x^2 = a + x$ which is a quadratic which can be solved for x, i.e. $x = \frac{1 + \sqrt{1 + 4a}}{2}$

#3 TRUE/FALSE. The easiest question. No one seemed to notice that $\lim_{n\to\infty} \frac{p_n - p}{q_n - q} = 0$ is the definition of saying that $q_n = o(p_n)$. In other words, q_n must converge faster than p_n, which we knew since q_n is quadratically convergent, and p_n is merely linear. So the statement is always TRUE. Obviously the machine precision depends on how much memory the calculating device you perform the operation $1 + \epsilon_m$ allocates for floating point numbers, especially the mantissa. This has NOTHING to do with underflow, however, it is merely a question of the "size of the holes" in the floating point number line. You should have remembered the different answers we got in class for the machine precisions of different TI calculators to say that the statement is FALSE.

#4 Root-finding methods. Look at Method B! at $n = 8$ it has $f(2.40482556) = 0.000e + 000$. Surely that means that it has found the root to the machine precision of the computing device. So you know 8 digits of accuracy. $z_{01} = 2.40482556 \approx 2.4048256$ to 7 decimal places. The error in $|x_n - x_{n-1}|$ is $\mathcal{O}(10^{-12})$ so you know those first 8 digits are completely accurate. For part (b) clearly Method A has to be bisection: look at the size of $|x_n - x_{n-1}|$. It is *exactly* 2^{-n}! Method B must be faster than linear convergence (i.e. superlinear). It converges so fast you might think quadratic convergence, but that would mean Newton's Method and how the heck are you going to differentiate $J_0(x)$? It must be Secant or False-Position (turns out to be Secant, but I don't think there's anyway you can tell just from the data). For part(c) the algorithms do not halt until the rightmost column goes beneath $5.00e - 008$ so it is either Criteria 2 or Criteria 3. If it was Criterial 1 or Criteria 3 then the algorithms would stop much earlier once the $f(x_n)$ value was small, regardless of what was happening with $|x_n - x_{n-1}|$. Part (d) is just a give away: can you read the data in the table? Clearly Criteria 1 & 4 must give the same number of iterations, as must Criteria 2 & 3 for this problem.