Test 1: Numerical Analysis

Math 370 Monday October 16, 2000

Name: ____________________

Directions: Read *all* 4 (four) problems first before answering any of them. This is a one hour, open-notes, open book, test. You must show all relevant work to support your answers. Use complete English sentences and indicate your final answer from your "scratch work."

No.	Score	Maximum
1		20
2		25
3		20
4		35
Total		100

1. *[20 points total.]* **Order Notation and Taylor Series.**

Describe the behavior of the given function $F(h)$ below for very small values of h, i.e. $|h| << 1$, or as $h \to 0$.

$$F(h) = \cos(h) + \cosh(h)$$

[Write your answer in the form $F(h) = \alpha + \mathcal{O}(h^p)$]

Check your answer using the limit definition of "big oh."

[NOTE: $\sinh(x) = \dfrac{e^x - e^{-x}}{2}, \qquad \cosh(x) = \dfrac{e^x + e^{-x}}{2}$]

2. *[25 points total.]* **Sequences and Limits.**

(a) *[10 pts.]* Consider the sequence $p_n = \dfrac{1}{n^n}$. Show that the asymptotic order of convergence of p_n to its limit p is **superlinear**.

(b) *[5 pts.]* Consider the sequence $q_n = \dfrac{1}{10^{2n}}$ show that the asymptotic order of convergence of q_n to its limit q is **linear**.

(c) *[10 pts.]* Consider the sequence $r_n = \dfrac{1}{10^{2^n}}$. Show that the asymptotic order of convergence of r_n to its limit r is **quadratic**.

3. *[20 points total.]* **Rates of Convergence.** Let's compare the relative rate of convergence of the sequences from **2.**

(a) *[15 pts.]* Given the tolerance $\epsilon = 10^{-6}$ compute out how many steps it takes for the sequence to get within ϵ of its limit for each of the sequences $\{p_n\}$, $\{q_n\}$ and $\{r_n\}$.

(i) $|p_{NP} - p| \leq \epsilon$ [HINT: Look at Question 4 to help you answer this question]

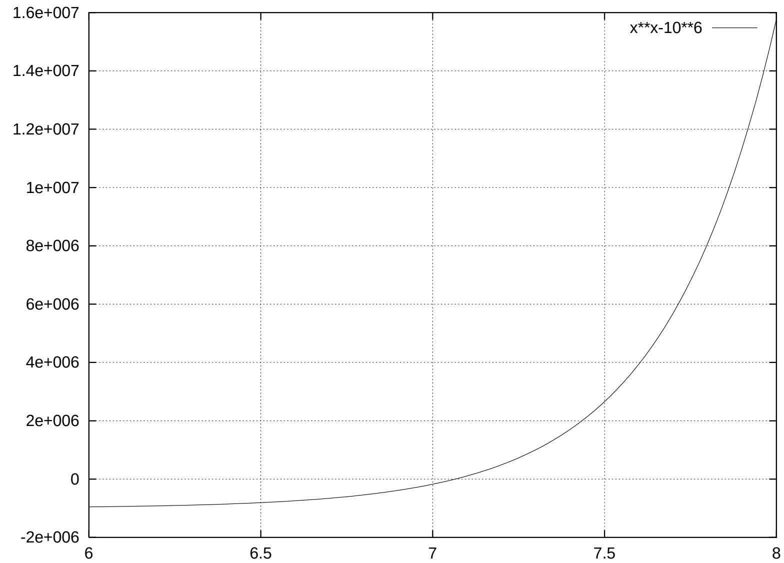
(ii) $|q_{NQ} - q| \leq \epsilon$

(iii) $|r_{NR} - r| \leq \epsilon$

(b) *[5 pts]* Explain the relative sizes of NP, NQ and NR.

4. *[35 pts. total]* **Root-Finding.**

(a) *[15 pts]* Given the graph of $f(x) = x^x - 10^6$ below, what method would you use to find r, the exact input value which results in $f(r)$ being identically zero? Discuss why your method is better than other options you rejected. State your initial guess (or bracket) you would use and explain why you chose the method you did.



(b) *[10 pts]* Use the method and initial guess (or bracket) you selected from part (a) and execute enough iterative steps to determine r to one decimal place. (You may want to indicate your estimates on the graph.) You can use the computers to assist you with this problem, if you wish.

(c) *[10 pts]* Suppose your tolerance for error in the solution of the problem is 0.05 and your numerical method produces a sequence of approximations $\{x_n\}$. What convergence criteria for your numerical method would result in the most accurate answer for r. Why?

Criteria 1: $|f(x_n)| \leq 0.05$

Criteria 2: $|x_n - x_{n-1}| \leq 0.05$

Criteria 3: $|f(x_n)| \leq 0.05$ AND $|x_n - x_{n-1}| \leq 0.05$

Criteria 4: $|f(x_n)| \leq 0.05$ OR $|x_n - x_{n-1}| \leq 0.05$

Rank the convergence criteria in order of decreasing accuracy.