Question 1

If we are told that the general solution to the linear homogeneous system $Y' = AY$ is $Y = c_1 e^{-4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then an equivalent form of the solution is

(a) $y_1 = -2c_1 e^{-4t} + c_1 e^{-4t}$ and $y_2 = 2c_2 e^{3t} + 3c_2 e^{3t}$

(b) $y_1 = -2c_1 e^{-4t} + 2c_2 e^{3t}$ and $y_2 = c_1 e^{-4t} + 3c_2 e^{3t}$

(c) $y_1 = -2c_1 e^{-4t} + c_1 e^{3t}$ and $y_2 = 2c_2 e^{-4t} + 3c_2 e^{3t}$

(d) All of the above.

(e) None of the above.

Question 2

If $Y = e^{-2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a solution to the linear homogeneous system $Y' = AY$, which of the following is also a solution?

(a) $Y = 2e^{-2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $Y = 3e^{-2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} - 4e^{-4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $Y = 1/4e^{-4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) All of the above

(e) None of the above

Question 3

The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. In the long term, phase trajectories:

(a) become parallel to the vector $v_2 =< -2, 1 >$.

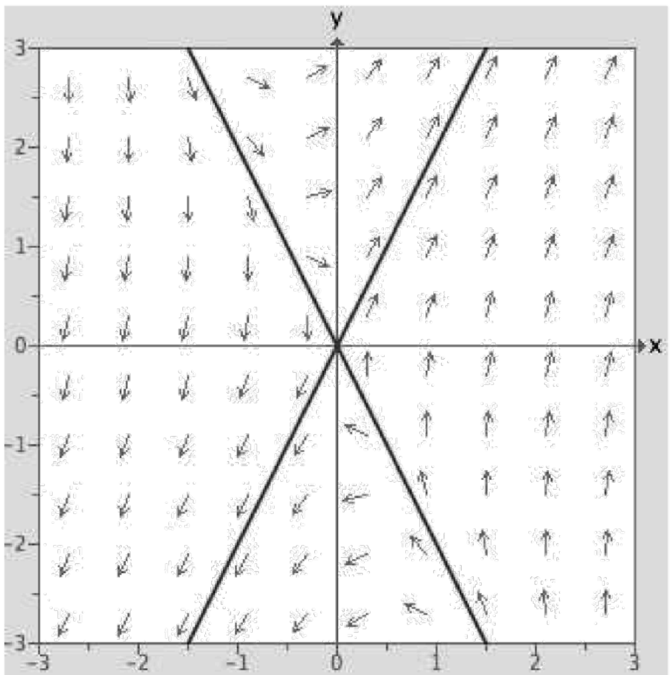
(b) tend towards positive infinity.

(c) become parallel to the vector $v_1 =< 1, 2 >$.

(d) tend towards 0.   (e) None of the above

Question 4

The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_2 is

(a) positive real

(b) negative real

(c) zero

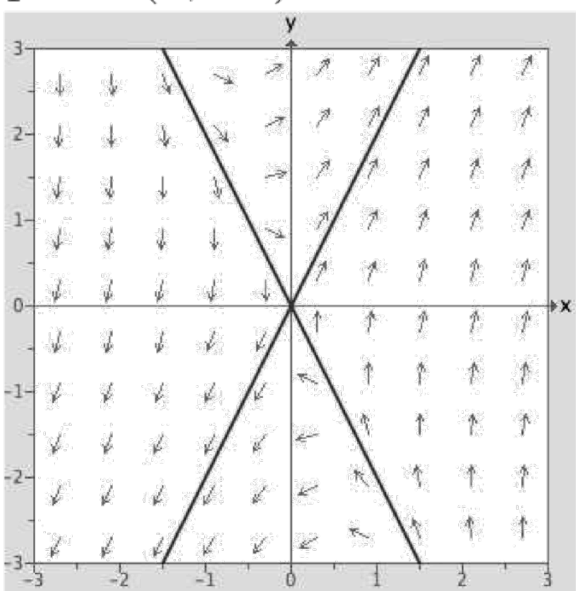
(d) complex

(e) There is not enough information

Question 5

The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.

Suppose we have a solution $\vec{Y}(t)$ to this system of differential equations which satisfies initial condition $\vec{Y}(t) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, -2)$. Which statement best describes the behavior of the solution as $t \to \infty$?



(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 2 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< 1, -2 >$.

(d) The solution spirals and returns to the point (x_0, y_0).

(e) There is not enough information.

Question 6

Which phase portrait below corresponds to the linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively?