Differential Equations

Math 341 Fall 2013 MWF 12:50-1:45pm Fowler 307

 `http://faculty.oxy.edu/ron/math/341/13/`

Worksheet 13

TITLE Euler's Method for Systems of ODEs
CURRENT READING Blanchard, 2.5

Homework Assignments due Friday October 11
Section 2.2: 7, 8, 11, 21* (EXPLAIN!), 24, 26.
Section 2.4: 2, 5, 7, 8.
Section 2.5: 2, 3.
Chapter 2 Review: 2, 3, 7, 12, 13 15, 16, 20, 30*

SUMMARY
It's baaack! We'll look at how to use Euler's Method for estimating solutions to systems of ODEs, i.e. $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x})$.

1. Euler's Method for Systems

The algorithm for generating approximate solutions to the ODE $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x})$ with initial condition $\vec{x}(0) = \vec{x_0}$ is

$$\vec{x}_{new} = \vec{x}_{old} + \vec{F}(\vec{x}_{old})\Delta t$$

EXAMPLE
A lot of the time the systems we will be looking at are systems of two ODEs, so in the case the IVP looks like

$$\begin{aligned}\frac{dx}{dt} &= f(x,y), \qquad x(0) = x_0\\ \frac{dy}{dt} &= g(x,y), \qquad y(0) = y_0\end{aligned}$$

The Euler's Method algorithm for a system of two ODEs looks like

$$\begin{aligned}x_{new} &= x_{old} + f(x_{old}, y_{old})\Delta t\\ y_{new} &= y_{old} + g(x_{old}, y_{old})\Delta t\end{aligned}$$

Exercise
Conside the system $\frac{dx}{dt} = x + y$; $\frac{dy}{dt} = 4x - 2y$. Starting at $(x, y) = (1, 0)$ and $\Delta t = 0.5$ let's take two "Euler steps" to approximate the solution curve through this point.

In *Worksheet #10* we were introduced to the Lotka-Volterra model of predator-prey populations.

$$\begin{aligned}\frac{dR}{dt} &= 2R - 1.2RF \\ \frac{dF}{dt} &= -F + 0.9RF\end{aligned}$$

$\boxed{\text{GROUPWORK}}$

Let's use Euler's Method with a $\Delta t = 1$ and the table below to estimate the population of rabbits and foxes after 3 time-steps, starting with $R(0) = 1$, $F(0) = 1$

t	R	F	R'	F'	ΔR	ΔF	Δt

Clearly, the most efficient way to do this would be to use a computer. Go to the computers and look at the spreadsheet `PredatorPrey.xls` on the S-drive and verify (and extend) your calculations.