Differential Equations

Math 341 Spring 2005 — MWF 8:30 - 9:25am Fowler North 2

 — http://faculty.oxy.edu/ron/math/341

Class 12: Monday February 14

TITLE *Systems of First Order DEs*
CURRENT READING Zill, 3.3

SUMMARY
We will explore models which are described using systems of first-order differential equations and indicate the complex dynamics which can occur.

1. Systems of n First-Order Differential Equations as n-th Order DEs

The general form for a system of first order DEs is:

$$\frac{dx}{dt} = f_1(x, y, t)$$

$$\frac{dy}{dt} = f_2(x, y, t)$$

If f_1 and f_2 are linear in both variables x and y then the system is called **linear**, otherwise the system is called **nonlinear**.

Note that the theory of n linear first-order systems of DEs is inextricably linked to the theory of linear n-th order DEs. For example, if $y'' = f(t, y(t)), y'(t)$ and one makes the substitution $u_2 = y'(t)$ and $u_1 = y(t)$ this second-order DE can be converted into a system of 2 first-order DEs.

Exercise Show that the third-order equation $y^{(3)} + 3y'' + 2y' - 5y = \sin(2t)$ can be written as a linear system of first-order DEs.

2. 3-species decay model

Consider a radioactive decay series where element X decays into element Y at a rate λ_1 which then decays into element Z at a rate λ_2. This is not an uncommon occurrence particularly in the case of Uranium.

The differential equations modelling this situation are:

$$\begin{aligned} \frac{dx}{dt} &= -\lambda_1 x \\ \frac{dy}{dt} &= \lambda_1 x - \lambda_2 y \\ \frac{dz}{dt} &= \lambda_2 y \end{aligned}$$

where $x(t)$, $y(t)$ and $z(t)$ model the amount of each species X, Y and Z at time t. If initially at $t = 0$, $x = x_0$, $y = y_0$ and $z = z_0$ how would you solve the system?

3. Lotka Volterra Model

Probably the most famous system of ordinary differential equations is the **Lotka-Volterra predator-prey model**. (The S-I-R model of epidemics would be a close second.) $x(t)$ denotes the population of predators (foxes) and $y(t)$ denotes the population of their prey (rabbits). Let a, b, c and d be positive parameters; the model is represented by:

$$\begin{aligned} \frac{dx}{dt} &= -ax + bxy \\ \frac{dy}{dt} &= -cxy + dy \end{aligned}$$

For what values of x and y are there constant solutions? (in other words, find the stationary points of the system of DEs.)

Using the CD-rom, explore the solution curves for different initial conditions if a=0.16, b=0.08, c=0.9 and d=4.5. Describe what you see. What happens as you approach the initial guess $x(0) = 5, y(0) = 2$?