# Differential Equations

Math 341 Spring 2005 — MWF 8:30 - 9:25am Fowler North 4

 — http://faculty.oxy.edu/ron/math/341

## *Class 6*: **Monday January 31**

**TITLE** First-Order Linear DEs
**CURRENT READING** Zill, 2.3

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**Homework Set #3**
Zill, Section 2.2: 2*, 3, 4*, 7, 9*, 16, 17*, 23*, 25, 26 *EXTRA CREDIT 31, 39, 44*
Zill, Section 2.3: 3, 4, 7*, 9, 15*, 22*, 29, 30, 31, 34* *EXTRA CREDIT 35, 43, 50*
Zill, Section 2.4: 5, 10*, 11*, 19, 27*, 30, 32, 38* *EXTRA CREDIT 42, 44*

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**SUMMARY**
We will be introduced to another technique for finding the solution of a DE. This time we restrict the class of DEs we are trying to solve to be first-order **linear** ordinary differential equations.

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### 1. Initial Value Problems

DEFINITION: **first-order linear DE**

A **first-order linear DE** has the form $a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$. When $g(x) = 0$ the equation is called **homogeneous**, when otherwise the the DE is called **nonhomogeneous**.

The standard form of a nonhomogeneous first-order linear DE is

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{1}$$

The general solution of this DE, $y(x)$ can be written as the sum of two solutions, $y_h(x)$ which solves the homogeneous version of the standard form (i.e. $Q(x) = 0$), and $y_p(x)$ which is a particular solution of the nonhomogeneous form of the DE. In other words, $y(x) = y_h(x) + y_p(x)$. (Note, the book uses the symbol $y_c(x)$ instead of $y_h(x)$.)

**Proof**
It's fairly straightforward to prove that this general solution can be split into homogeneous and non-homogeneous parts:

It's also straightforward to use separation of variables to produce an expression for the solution to the homogeneous form, $y_h(x)$:

**Integrating Factor**

It turns out that if one takes the function $\mu(x) = e^{\int P(x)dx}$ and multiplies each term in the standard form in (1) by this integrating factor one obtains:

$$\begin{aligned} e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y &= Q(x)e^{\int P(x)dx} \\ \frac{d}{dx}\left(e^{\int P(x)dx}y\right) &= Q(x)e^{\int P(x)dx} \\ e^{\int P(x)dx}y &= \int Q(x)e^{\int P(x)dx}\,dx \\ y(x) &= e^{-\int P(x)dx}\int Q(x)e^{\int P(x)dx}\,dx \end{aligned}$$

EXAMPLE **Zill, page 61, Example 3.** Solve $xy' - 4y = x^6e^x$

Exercise **Zill, page 62, Example 5.** Solve $\dfrac{dy}{dx} + y = x, \quad y(0) = 4.$

EXAMPLE **Zill, page 63, Example 6.**

Solve $\dfrac{dy}{dx} + y = f(x), \quad y(0) = 0,$ where $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$

NOTE: the graph of the function $y(x)$ on page 63 but also think about the question on page 66 in **Exercise 42**: Why is it technically incorrect to say that the function graphed is a solution of the IVP on the interval $[0, \infty)$?