Differential Equations

Math 341 Spring 2005 — MWF 8:30 - 9:25am Fowler North 4

 — http://faculty.oxy.edu/ron/math/341

Class 5: **Friday January 28**

TITLE The Method of Separation of Variables
CURRENT READING Zill, 2.2 and 2.3

Homework Set #3
Zill, Section 2.2: 2*, 3, 4*, 7, 9*, 16, 17*, 23*, 25, 26 *EXTRA CREDIT 31, 39, 44*
Zill, Section 2.3: 3, 4, 7*, 9, 15*, 22*, 29, 30, 31, 34* *EXTRA CREDIT 35, 43, 50*
Zill, Section 2.4: 5, 10*, 11*, 19, 27*, 30, 32, 38* *EXTRA CREDIT 42, 44*

SUMMARY
We will be introduced to our first technique for finding the solution of a DE. We restrict the class of DEs we are trying to solve to be first-order "separable" ordinary differential equations.

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1. Solving Separable Differential Equations

DEFINITION: **separable DE**

A **separable first-order differential equation** is one which has the form $\frac{dy}{dx} = g(x)h(y)$

The technique for solution is to separate the variables in the equation by placing everything with an independent variable on one side, and everything with a dependent variable on the other. This produces:

$$\frac{dy}{h(y)} = g(x)dx$$

One can then treat each side of the equation as an indefinite integral,

$$\int \frac{dy}{h(y)} = \int g(x)dx$$

which, if each function $1/h(y)$ and $g(x)$ have anti-derivatives $H(y)$ and $G(x)$, respectively produces

$$H(y) = G(x) + C$$

The above equation thus defines (implicitly) a family of solutions to the given first-order DE. When an initial condition $y(a) = b$ is also given, then a particular solution can be obtained.

Exercise **Zill, page 54, #25.** $x^2\frac{dy}{dx} = y - yx, \quad y(-1) = 1.$

EXAMPLE Consider again the IVP $\frac{dy}{dx} = xy^{1/2}, \quad y(0) = 0$. We can show that $y = 0$, $y = \left(\frac{x^2}{4} + c\right)^2$ and $y(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{16}(x^2 - 1)^2, & x \geq 1 \end{cases}$ are **all** solutions of this IVP, although only one of these is derived using the method of separation of variables.

Where Singular Solutions Come From

The moral of the story is that you need to be careful when solving a problem using separation of variables, because if $y = A$ is a constant solution of the separable differential equation $y' = g(x)h(y)$ (i.e. $h(A) = 0$) then using separation of variables may not include this constant solution in the computed family of solutions, in other words, $y(x) = A$ may be a singular solution of the DE.

Exercise **Zill, page 52, Example 3.** Solve $\frac{dy}{dx} = y^2 - 4$.