Differential Equations

Math 341 Spring 2005 — MWF 8:30 - 9:25am Fowler North 4

 — http://faculty.oxy.edu/ron/math/341

Class 4: **Wednesday January 26**

TITLE Obtaining information from a DE without solving it
CURRENT READING Zill, 2.1 and 2.2

Homework Set #2
Section 1.2 # 1, 2*, 3, 4*, 5, 6*, 15, 17, 20*, *EXTRA CREDIT: 31, 32*
Section 1.3 # 1, 2, 3*, 5*, 8*, *EXTRA CREDIT: 13, 14*
Section 2.1:# 1, 2*, 3, 4*, 5, 6*, 8*, 12, 17*, 21, 22 *EXTRA CREDIT 19, 29, 32, 37*
EXTRA CREDIT HW Set: CHAPTER 1 REVIEW: # 5, 7, 8, 9, 10, 15, 16, 17, 21, 22

SUMMARY
We begin our analysis of first order DEs by discovering how much information about the solution one can obtain without being able to obtain an explicit formula for the solution itself.

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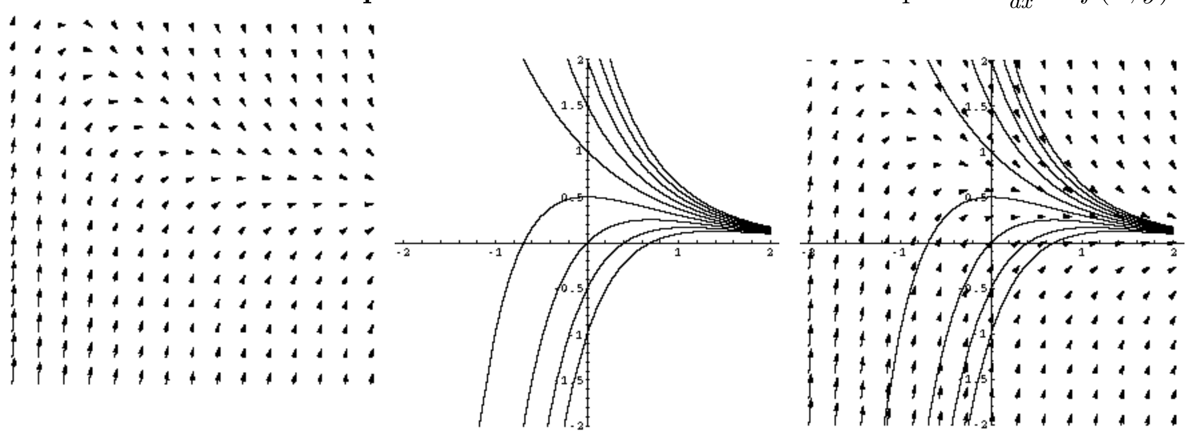
1. Direction Fields

DEFINITION: **direction field** or **slope field**

A collection of short, oriented line segments called lineal elements placed at each point (x, y) over a rectangular grid, which have slopes evaluated at each point to be $f(x, y)$ are called the **direction field** or **slope field** of the first order differential equation $\frac{dy}{dx} = f(x, y)$.



The above figure is a direction field for the differential equation $y' = e^{-x} - 2y$. Note, that you can use the direction field to produce solution curves of the differential equation.

RECALL

When the derivative dy/dx is positive (negative) on an interval I, the function $y(x)$ is increasing (decreasing) for all values of x on I.

Sketching direction fields is really time consuming but there are a lot of software programs out there to assist you. Check out the `Resources` tab at the Math 341 website as well as the CD-rom that comes with textbook.

2. Autonomous First-Order Differential Equations

DEFINITION: **autonomous DE**

A differential equation in which the independent variable does not explicitly appear is known as an **autonomous differential equation**. For example, a first order autonomous DE has the form $y' = f(y)$.

DEFINITION: **critical point**

A **critical point** of an autonomous DE $y' = f(y)$ is a real number c such that $f(c) = 0$. Another name for critical point is **stationary point** or **equilibrium point**. If c is a critical point of an autonomous DE, then $y(x) = c$ is a constant solution of the DE.

DEFINITION: **phase portrait**

A **one dimensional phase portrait** of an autonomous DE $y' = f(y)$ is a diagram which indicates the values of the dependent variable for which y is increasing, decreasing or constant. Sometimes the vertical version of the phase portrait is called a **phase line**.

EXAMPLE

Consider the autonomous differential equation $\frac{dy}{dt} = y(a - by)$.

1 Find the critical points of the DE.

2 Determine the values of y for which $y(t)$ is increasing and decreasing

3 Draw the vertical phase line for this DE

3. Classifying Critical Points: Stable, Unstable, Semi-Stable

A critical value c is a point where $y' = 0$ splits an interval into two different regions. So there are four possible scenarios for the behavior near c: $(+, 0, +)$, $(+, 0, -)$, $(-, 0, +)$ and $(-, 0, -)$.

EXAMPLE

Draw the phase line for each of these cases and then classify the corresponding critical points as asymptotically stable (i.e. attractor), unstable (repellor) or semi-stable.

Exercise

Zill, Question #25, Page 48. $y' = y^2(y^2 - 4)$. Find the critical points and phase portrait for the given autonomous DE. Sketch typical solution curves in the regions of the xy-plane determined by the graphs of the equilibrium solutions.