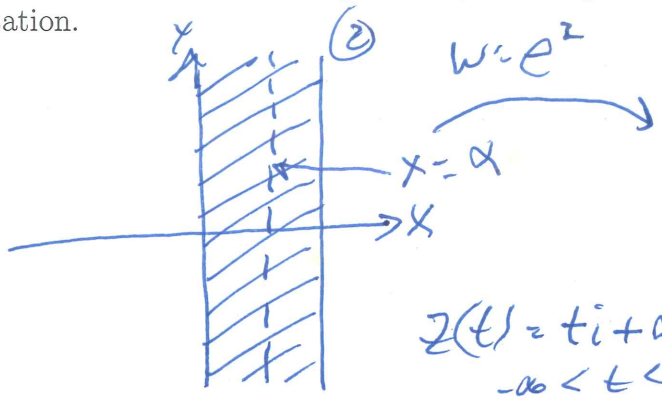
(1a) (*3 points*) What is the image D' of the set $D = \{z \in \mathbb{C} : 0 \leq \mathrm{Re}(z) \leq \pi\}$ under the mapping $w = e^z$? **Sketch the image and pre-image sets.** Write down a definition of D' in set notation.

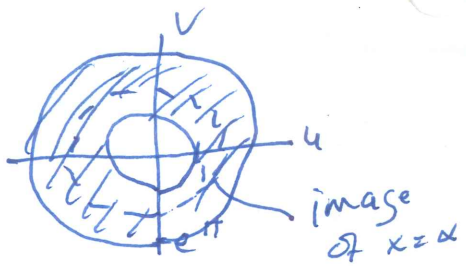


D

$z(t) = ti + \alpha$

$-\infty < t < \infty$

α is fixed but ranges from 0 to π

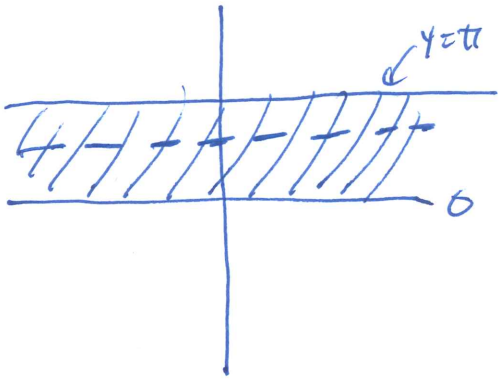


$w = e^{it} \cdot e^{\alpha}$

$-\infty < t < \infty$

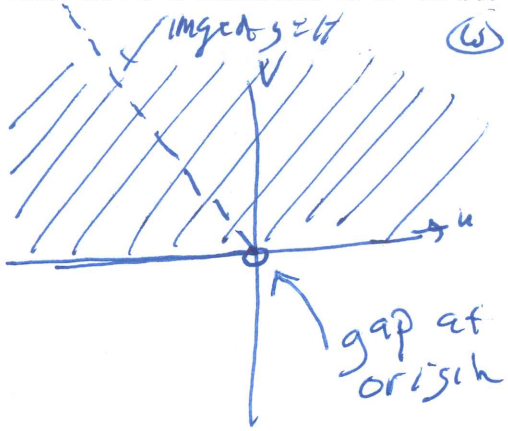
$D' = \{w \in \mathbb{C} : 1 \leq |w| \leq e^{\pi}\}$

(1b) (*3 points*) What is the image B' of the set $B = \{z \in \mathbb{C} : 0 \leq \mathrm{Im}(z) \leq \pi\}$ under the mapping $w = e^z$? **Sketch the image and pre-image sets.** Write down a definition of B' in set notation.



B

$w = e^z$



$z(t) = i\beta + t, \quad -\infty < t < \infty$

β ranges from 0 to π

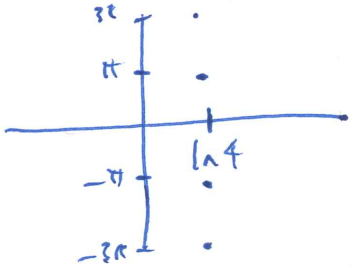
$w = e^{i\beta + t} = e^{i\beta} e^{t}, \quad -\infty < t < \infty$

$B' = \{w \in \mathbb{C} : \mathrm{Im}(w) \geq 0\} \setminus \{0\}$

1(c) (*4 points*) Find all solutions of $e^z = -4$ where $z \in \mathbb{C}$. Draw a picture inficating the location of the solution points in the complex plane.

$e^z = -4 = 4 \cdot -1 = e^{\ln 4} \cdot e^{\pi i + 2k\pi i}$

$z = \ln 4 + \pi i + 2k\pi i$



Infinite line of points

$\{z \in \mathbb{C} : z = \ln 4 + (2k+1)\pi i, \; k \in \mathbb{Z}\}$