Quiz 4 — Complex Analysis

Name: _____

ASSIGNED: **Friday February 12**
DUE: **Wednesday February 17**

Time Begun: _____
Time Ended: _____

Prof. Ron Buckmire

Topic : Harmonic Conjugates of Analytic Functions

The **learning goal** of this quiz is to demonstrate the usefulness of the Cauchy-Riemann equations and your understanding of Harmonic Functions

Reality Check:

EXPECTED SCORE : _____/**10** ACTUAL SCORE : _____/**10**

Instructions:

1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Wednesday February 17**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

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1. (*10 points*) We want to find a formula for an **entire** function $f(z)$ but all we know is that its real part is given by $u(x, y) = x^3 - 3xy^2 - 4xy + 6$ and that it maps the point $(1, 1)$ to the origin.

(a) (*6 points*) Use the Cauchy-Riemann Equations (i.e. $u_x = v_y, \quad u_y = -v_x$) to find the imaginary part of $f(z)$, sometimes written as $v(x, y)$, *exactly.*

(b) (*4 points*) Confirm that both $v(x, y)$ and its harmonic conjugate $u(x, y)$ are examples of functions of two variables $\phi(x, y)$ that satisfy the 2-dimensional Laplace Equation $\phi_{xx} + \phi_{yy} = 0$.