Math 312 Spring 2014 **SHOW ALL YOUR WORK** Quiz **SEVEN**

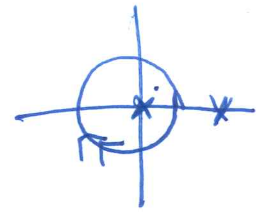
1. Consider the following contour integral and evaluate it for the various contours.

$$\oint_C \frac{3z+1}{z(z-2)^2}\,dz$$

(a) *(3 points.)* C is the contour $|z|=1$ traversed **twice** clockwise.

$$\oint_C \frac{\frac{3z+1}{(z-2)^2}}{z}\,dz = (-2)\cdot 2\pi i\cdot f(0)$$

$$= -2\cdot 2\pi i\cdot \frac{1}{4} = \boxed{-\pi i}$$

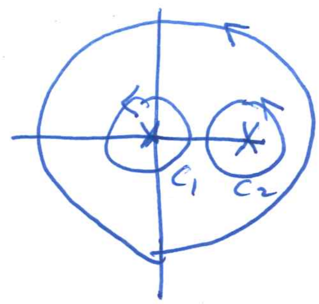


$$f(z)=\frac{3z+1}{(z-2)^2}$$

$$f(0)=\frac{1}{(-2)^2}=\frac{1}{4}$$

(b) *(3 points.)* C is the contour $|z|=3$ traversed once counter-clockwise.

$$\oint_C \frac{3z+1}{z(z-2)^2}\,dz = \oint_{C_1}\frac{\frac{3z+1}{(z-2)^2}}{z}\,dz + \oint_{C_2}\frac{\frac{3z+1}{z}}{(z-2)^2}\,dz$$



$$= 2\pi i\cdot\frac{1}{4} + 2\pi i\cdot\frac{g'(2)}{1!}$$

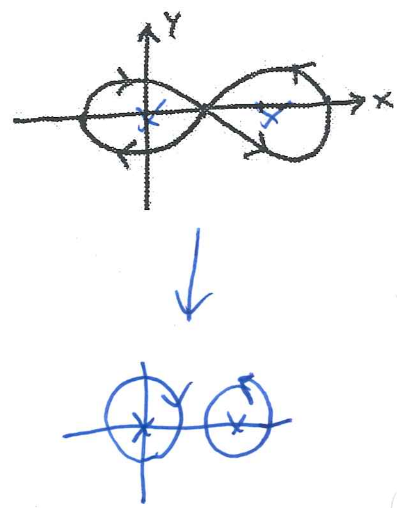
$$\oint \frac{f(z)\,n!}{(z-z_0)^{n+1}}\,dz = 2\pi i f^{(n)}(z_0)$$

$$= 2\pi i\cdot\frac{1}{4} + 2\pi i\left(-\frac{1}{4}\right) = \boxed{0}$$

$$g(z)=\frac{3z+1}{z}=3+\frac{1}{z} \qquad g'(z)=-\frac{1}{z^2}$$

(c) *(4 points.)* C is the contour shaped like the symbol ∞ intersecting the x-axis at the points $z=-1, z=1$ and $z=3$ and where the right segment is traversed once counter-clockwise and the left segment is traversed once clockwise.

$$I = \oint_C \frac{\frac{3z+1}{(z-2)^2}}{z}\,dz + \int_C \frac{\frac{3z+1}{z}}{(z-2)^2}\,dz$$



$$I = (-1)\cdot 2\pi i\cdot\frac{1}{4} + 2\pi i\cdot\frac{-1}{4}$$

$$I = -\frac{\pi i}{2} - \frac{\pi i}{2} = \boxed{-\pi i}$$