Math 312 Spring 2014 **SHOW ALL YOUR WORK** Quiz SIX

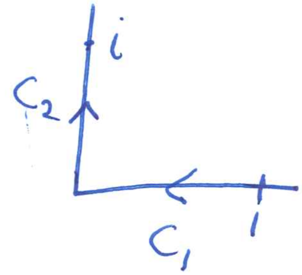
Consider the following integral in complex variables

$$\int_C (x^2 - y^2) + (2xy)i \, dz$$

(a) (*4 points*) Compute the value of the integral where C consists of line segments going from 1 to i by going along the x axis and then the y axis. Sketch the contour and write down the parametrization(s) used.

$C_1 : z(t) = 1 - t, \quad 0 \le t \le 1 \qquad z' = -1$

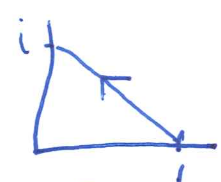
$C_2 : z(t) = it, \quad 0 \le t \le 1 \qquad z' = i$



$$\int_C x^2 - y^2 + 2xyi \, dz = \int_0^1 (1-t)^2(-1)\,dt + \int_0^1 -t^2 \cdot i \cdot dt$$

$$= \int_1^0 u^2 du + \left(-\frac{t^3}{3} i\right)\Big|_0^1 = -\frac{1}{3} - i\frac{1}{3}$$

(b) (*4 points*) Compute the value of the integral where C consists of one line segment going from 1 to i directly. Sketch the contour and write down the parametrization(s) used.



$z(t) = 1 - t + it$

$0 \le t \le 1$

$z' = -1 + i$

$$\int_0^1 \left[(1-t)^2 - t^2 + 2(1-t)t i\right](-1+i)\,dt$$

$$= (i-1)\int_0^1 (1 - 2t) + (2t - 2t^2)i \, dt$$

$$= (i-1)\left[t - t^2 + \left(t^2 - \frac{2t^3}{3}\right)i\right]\Big|_0^1$$

$$= (i-1)\left[1 - 1 + \left(1 - \frac{2}{3}\right)i\right] = (i-1)\frac{i}{3}$$

$$= -\frac{1}{3} - \frac{i}{3}$$

(c) (*2 points*) Do you get the same answer for evaluating this function along the two different contours? Do you expect to get the same answer? Give a short reason to support your answer(s). [HINT: is the function being integrated an analytic function? How do you know?]

$f(z) = z^2 = x^2 - y^2 + 2xyi$ is an analytic function!

Thus we expect $\int_1^i z^2 dz = \frac{z^3}{3}\Big|_1^i = -\frac{i}{3} - \frac{1}{3}$ regardless of path!