Adapted from **Zill & Shanahan, pg. 70, #33.**

1. (*10 points*) A **fixed point** of a mapping $w = f(z)$ is a point z_0 where $f(z_0) = z_0$.

(a) (*2 points*) Does the linear mapping $f(z) = az + b$ have a fixed point z_0? If so, find z_0 in terms of the values of a and b. (NOTE: a and b can be any number in the complex plane).

$f(z_0) = z_0 \Rightarrow a z_0 + b = z_0$

$b = z_0 - a z_0$

$b = z_0(1-a)$

$\frac{b}{1-a} = z_0 \quad (a \neq 1)$

(b) (*2 points*) Give an example of a complex linear mapping (i.e. choose values for a and b) so that $f(z)$ has **no fixed points**.

If $a=1$ then $f(z) = z+b$ will have no fixed points because it's a translation

(We're only considering FINITE fixed points. Technically ∞ is ALWAYS a fixed pt)

(c) (*2 points*) Give an example of a complex linear mapping (i.e. choose values for a and b) so that $f(z)$ has **more than one fixed point**. [HINT: There is only one such mapping possible!]

$a = 1 \quad b = 0$

$f(z) = z$ is the identity mapping,

so EVERY point in the plane is a fixed pt.

(d) (*4 points*) The inverse mapping $g(w) = cw + d$ is the complex linear mapping such that $g(f(z)) = z$ and $f(g(w)) = w$. In other words $z = g(w)$ "undoes" whatever the mapping $w = f(z)$ does. Find the values of c and d (in terms of the parameters a and b) so that $g(w) = cw + d$ is the inverse mapping of $f(z)$. Confirm that your choice for $z = g(w)$ is indeed the inverse of $w = f(z)$.

$w = az + b \Rightarrow w - b = az \Rightarrow \frac{w-b}{a} = z$

$g(w) = cw + d = \frac{w}{a} - \frac{b}{a} \qquad f^{-1}(w) = \frac{w-b}{a}$

$c = \frac{1}{a}$ & $d = -\frac{b}{a}$

$f(g(w)) = a\left(\frac{w-b}{a}\right) + b = w - b + b = w$ ✓

$g(f(z)) = g(az+b) = \frac{1}{a}(az+b) - \frac{b}{a}$

$= z + \frac{b}{a} - \frac{b}{a} = z$ ✓

[Inverse only exists if $a \neq 0$]