a. *(3 points).* **Confirm** that $\sqrt{-15-8i} = \pm(1-4i)$.

Show $(-15-8i) = [\pm(1-4i)]^2 = (1-4i)^2$

$= 1^2 - 4^2 + 2\cdot 1\cdot(-4i)$

$= 1 - 16 - 8i$

$= -15-8i$ ✓

b. *(4 points).* **Find** all the solutions of $z^2 + (2i-3)z + 5 - i = 0$ and write them in rectangular form $a+bi$. (HINT: You will probably want to use the result from (a) at some point).

use quadratic formula

$$z = \frac{-(2i-3) \pm \sqrt{(2i-3)^2 - 4\cdot 1\cdot(5-i)}}{2}$$

$$= \frac{-2i+3 \pm \sqrt{-4+9-12i-20+4i}}{2}$$

$$= \frac{-2i+3 \pm \sqrt{-15-8i}}{2}$$

$$= \frac{-2i+3 \pm (1-4i)}{2} = \frac{-2i+3+(1-4i)}{2} = \frac{2-6i}{2} = 1-3i$$

or

$$\frac{-2i+3-1+4i}{2} = \frac{2i+2}{2} = 1+i$$

c. *(3 points).* **Show** that $z_1 = 2-3i$ and $z_2 = 1+i$ each solve the quadratic equation in (b) and write each one in exponential form.

$(1+i)^2 + (2i-3)(1+i) + 5 - i \stackrel{?}{=} 0$

$1+2i-1+2i-2-3-3i+5-i \stackrel{?}{=}$

$0 \stackrel{\checkmark}{=} 0$

$(2-3i)^2 + (2i-3)(2-3i) + 5 - i \stackrel{?}{=} 0$

$5-12i+4i-6+6+9i+5-i \stackrel{?}{=} 0$

$0 \stackrel{\checkmark}{=} 0$

$1+i = |1+i|e^{i\,\mathrm{Arg}(1+i)}$

$= \sqrt{2}e^{i\pi/4}$

$2-3i = |2-3i|e^{i\,\mathrm{Arg}(2-3i)}$

$= \sqrt{13}e^{i\theta}$

where

$\tan\theta = \frac{-3}{2}$ with θ in 2^{nd} quadrant

$\frac{\pi}{2} < \theta < \pi$