Math 312 Spring 2014 **SHOW ALL YOUR WORK** Quiz One

1. (*3 points*) Prove the following statements are true for any complex number z:
$\mathrm{Re}(z) = \dfrac{z+\bar{z}}{2}$ and $\mathrm{Im}(z) = \dfrac{z-\bar{z}}{2i}$

let $z = a+ib$

$\bar{z} = a-ib$

$\dfrac{z+\bar{z}}{2} = \dfrac{a+ib+a-ib}{2}$

$= \dfrac{2a}{2}$

$= a = \mathrm{Re}(z)$ □

$z-\bar{z} = \dfrac{a+ib-(a-ib)}{2i}$

$= \dfrac{2ib}{2i}$

$= b$

$= \mathrm{Im}(z)$ □

2. (*4 points*) Describe and sketch the set of points in the complex plane which satisfy the equation

$$\mathrm{Re}(z) + 1 = |z-1|$$

let $z = x+iy$

$\mathrm{Re}(z)+1 = |z-1|$

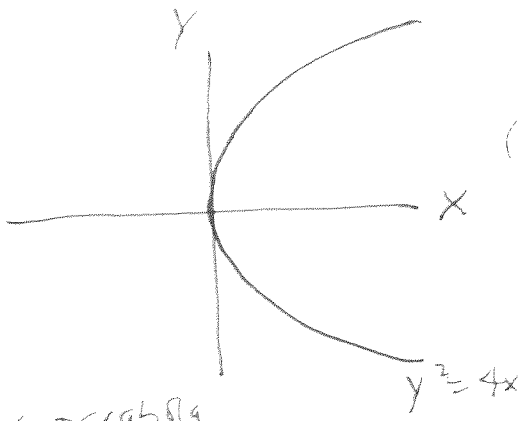
$x+1 = \sqrt{(x-1)^2+y^2}$

$(x+1)^2 = (x-1)^2+y^2$

$(x+1)^2-(x-1)^2 = y^2$

$(x+1+x-1)(x+1-(x-1)) = y^2$

$(2x)(2) = y^2 \Rightarrow y^2 = 4x$ parabola



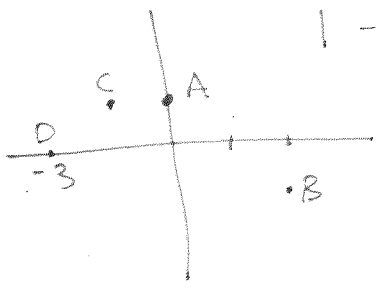
3. (*3 points*) In the complex plane, which of the points i, $2-i$, $-1+i$ and -3 is farthest from the origin? Which is closest? A B C D

$|i| = 1$

$|2-i| = \sqrt{2^2+1^2} = \sqrt{5}$

$|-1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

$|-3| = 3$



Distance from origin

D > B > C > A