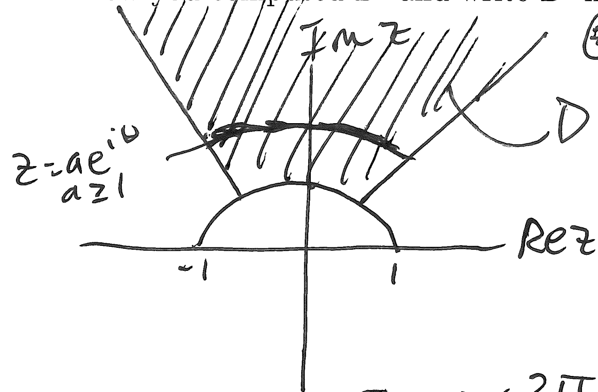
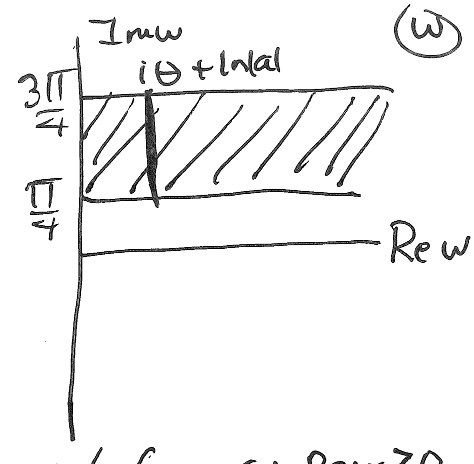
SHOW ALL YOUR WORK
& EXPLAIN EVERY ANSWER

1. (*3 points*) What is the image D' of the set $D = \{z \in \mathbb{C} : |z| \geq 1 \cap \pi/4 \leq \mathrm{Arg}(z) \leq 3\pi/4\}$ under the mapping $w = \mathrm{Log}(z)$? **Sketch the image and pre-image sets.** Remember to show the details how you computed D' and write D' in set-builder notation.



$w = \mathrm{Log}(z)$



$z = ae^{i\theta}, \ \frac{\pi}{4} < \theta < \frac{3\pi}{4}$

$a \geq 1$

$D' = \{w \in \mathbb{C} : \mathrm{Re}\,w \geq 0 \cap \frac{\pi}{4} < \mathrm{Im}\,w < \frac{3\pi}{4}\}$

$w = \mathrm{Log}\,z = \mathrm{Log}(ae^{i\theta})$

$= \mathrm{Ln}|ae^{i\theta}| + i\,\mathrm{Arg}(ae^{i\theta})$

$= \ln(a) + i\theta \qquad a \geq 1 \Rightarrow u \geq \ln 1 \Rightarrow u \geq 0$

$= u + iv \qquad \frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4} \Rightarrow \frac{\pi}{4} \leq v \leq \frac{3\pi}{4}$

2. (*1 point*) Find all solutions of $e^{e^z} = 1$. **Indicate the location of the solution(s) z in a sketch of the complex plane.**

$e^{e^z} = 1 = e^{0 + 2k\pi i}$

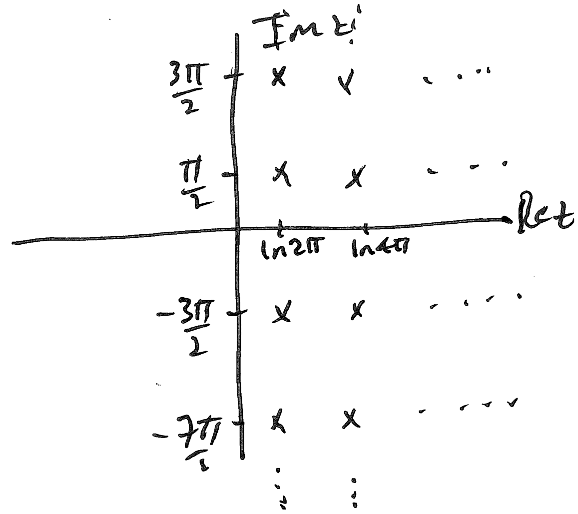
$e^z = 2k\pi i, \ k \in \mathbb{Z}$

$z = \log(2k\pi i)$

$= \ln|2k\pi i| + i\arg(2k\pi i)$

$= \ln|2k\pi| + i\frac{\pi}{2} + 2\ell\pi i$

$k \neq 0 \qquad \ell \in \mathbb{Z}$



3. (*1 point*) Give an example of an integer n and complex number z_1 such that $\mathrm{Log}((z_1)^n) \neq n\mathrm{Log}(z_1)$. **Show that alternate sides of the expression are not equal for the numbers you select.**

$(\mathrm{Log}\,z_1)^n \neq n\,\mathrm{Log}\,z_1$ for any z_1

$\mathrm{Log}\,z_1^n = n\,\mathrm{Log}\,z_1$ for some z not all

Let $z_1 = i, \ n = 4$

$z_1^n = i^4 = 1$

$\mathrm{Log}\,i^4 \stackrel{?}{=} 4\,\mathrm{Log}\,i$

$= \mathrm{Log}\,1 \stackrel{?}{=} 4(\ln 1 + i\frac{\pi}{2})$

$= 0 \qquad = 2\pi i$

$0 \neq 2\pi i$