

Nonlinear PDE's in Superconductivity

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Abstract

Superconductors produce the Meissner effect, where external magnetic fields are repelled to the surface of the material. There are two types of superconductors, type I and type II. The type II superconductor has a mixed state where the magnetic fields are allowed to exist in vortices within the material in a specific way so they do not destroy superconductivity. However, the type I superconductor does not have this mixed state. The goal of this project is describe the vortices of type II superconductors using partial differential equations, or PDE's. One particular equation comes from the limit of the Ginzburg-Landau equations.

Summary

Let the function $\underline{r}(t) = (x, Y(x, t), Z(x, t))$ be given. Then the first and simplest PDE is the following:

$$\frac{\partial Z}{\partial t} = \frac{1}{4\pi} \frac{\partial^2 Z}{\partial x^2} - \frac{1}{2\pi} K_1(2Z) + H_0 e^{-Z} \quad (1)$$

Where H_0 is a constant magnetic field and K_1 is the Bessel function of the second kind. This equation is derived from the limit of the Ginzburg-Landau equations, more formally known as the London equations.