Fourier Analysis and Digital Image Processing

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Introduction

Abstract

Fourier Analysis is the study of the way functions may be expressed or approximated by sums of much simpler trigonometric functions, and is an incredibly useful tool in image processing. We can think of each individual pixel in a digital image as points in the spatial domain to which we can apply an appropriate Fourier Transform, resulting in an image in the frequency domain. Once in the frequency domain, we can perform operations that will result in the desired manipulation of the image once converted back to the spatial domain using an inverse Fourier Transform. I will discuss the mathematics behind the Fourier Transform with regards to digital image processing, as well as explain the way in which operations in the frequency domain affect the corresponding image in the spatial domain.

Overview

- Fourier Transforms convert an image from the spatial domain to the frequency domain.
- Inverse Fourier Transforms convert an image from the frequency domain to the spatial domain.
- Since we only need a set of frequencies large enough to describe the image in the spatial domain, we use a Discrete Fourier Transform.
- The number of frequencies in the set is equal to the number of pixels in the spatial domain image. Thus, the image is the same size in each domain.
- We are able to manipulate the spatial domain image using certain operations in the frequency domain image.

Discrete Fourier Transform

For a square $N \times N$ image, the two-dimensional Discrete Fourier Transform is given by:

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j) e^{-i\pi(\frac{ki}{N} + \frac{lj}{N})}$$

And the corresponding inverse Fourier Transform is given by:

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l) e^{i2\pi(\frac{ka}{N} + \frac{lb}{N})}$$

These equations are separable into two one-dimensional Discrete Fourier Transforms.

Spatial Domain and Frequency Domain Images

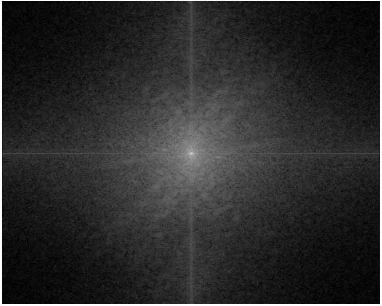
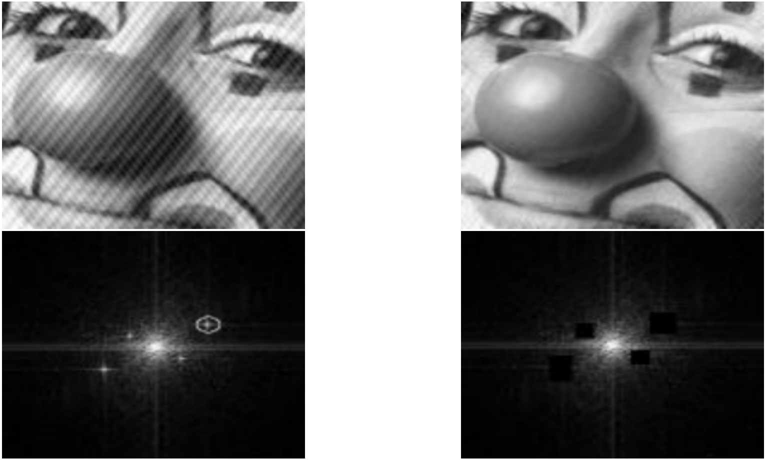


Image Processing in the Frequency Domain



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